Y	Q	P	RECURRENCE
15	3	2	A version of the following problem first appeared in print in the 16th Century. A frog and a toad fall to the bottom of a well that is 50 feet deep. Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well. The toad climbs 13 feet each day before resting. Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well. Their progress can be modelled by the recurrence relations: • $f_{n+1} = \frac{1}{3} f_n + 32, \qquad f_1 = 32$ • $t_{n+1} = \frac{3}{4} t_n + 13, \qquad t_1 = 13$ where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in. (a) Calculate t_2, the height of the toad at the end of the second day. **1** (b) Determine whether or not either of them will eventually escape from the well. **5**
16	3	1	A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3} u_n + 10$ with $u_3 = 6$. (a) Find the value of u_4. **1** (b) Explain why this sequence approaches a limit as $n \to \infty$. **1** (c) Calculate this limit. **2**
17	9	1	A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant. (a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2** (b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1** (ii) Calculate this limit. **2**
17	8	2	Sequences may be generated by recurrence relations of the form $u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$. (a) Show that $u_2 = 5k^2 - 20k - 20$. **2** (b) Determine the range of values of k for which $u_2 < u_0$. **4**

18	7	2	(a) (i) Show that $(x-2)$ is a factor of $2x^3-3x^2-3x+2$. **2** (ii) Hence, factorise $2x^3-3x^2-3x+2$ fully. **2** The fifth term, u_5, of a sequence is $u_5 = 2a-3$. The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$. (b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$. **1** For this sequence, it is known that • $u_7 = u_5$ • a limit exists. (c) (i) Determine the value of a. **3** (ii) Calculate the limit. **1**
19	4	1	A sequence is generated by the recurrence relation $$u_{n+1} = mu_n + c,$$ where the first three terms of the sequence are 6, 9 and 11. (a) Find the values of m and c. **3** (b) Hence, calculate the fourth term of the sequence. **1**
19	4	2	In a forest, the population of a species of mouse is falling by 2·7% each year. To increase the population scientists plan to release 30 mice into the forest at the end of March each year. (a) u_n is the estimated population of mice at the start of April, n years after the population was first estimated. It is known that u_n and u_{n+1} satisfy the recurrence relation $u_{n+1} = au_n + b$. State the values of a and b. **1** The scientists continue to release this species of mouse each year. (b) (i) Explain why the estimated population of mice will stabilise in the long term. **1** (ii) Calculate the long term population to the nearest hundred. **2**