2006 Paper 1

1a) $M(3,5)$ $\quad m_{BD} = \dfrac{5-(-5)}{3-(-2)} = 2$

$y-(-5) = 2(x+2)$

$y+5 = 2x+4$

$\underline{\underline{y = 2x-1}}$

b) $m_{BC} = \dfrac{-2+5}{7+2} = \dfrac{3}{9} = 1/3$

If ⊥, $m_1m_2 = -1$

$1/3 \times (-3) = -1$

at $A(-1,12)$

$y-12 = -3(x+1)$

$y-12 = -3x-3$

$\underline{\underline{y = -3x+9}}$

c) $2x-1 = -3x+9$ $\qquad y = 2(2)-1$

$5x = 10$ $\qquad = 4-1$

$\underline{\underline{x=2}}$ $\qquad \underline{\underline{y=3}}$

pt of intersection $\underline{\underline{(2,3)}}$

2a) $d_{CP} = \sqrt{(3-6)^2+(-2-1)^2}$

$= \sqrt{(-3)^2+(-3)^2}$

$\underline{\underline{r = \sqrt{18}}}$ $\qquad C(-2,3)$

$(x+2)^2+(x-3)^2 = 18.$

b) on x-axis, $y=0$

$(x+2)^2+(0-3)^2 = 18$

$x^2+4x+4+9 = 18$

$x^2+4x-5 = 0$

$(x+5)(x-1) = 0$

$\underline{x=1}$, $\underline{x=-5}$

$Q(-5,0)$. $\quad P(1,6)$

$m_{QP} = \dfrac{0-6}{-5-1} = \dfrac{-6}{-6} = 1$

If ⊥, $m_1m_2 = -1$

$\underline{m_{tgt} = -1}$

at Q: $y-0 = -1(x+5)$

$\underline{\underline{y = -x-5}}$

3a) $f(x) = 2x+3$ $\qquad g(x) = 2x-3$

$f(g(x)) = f(2x-3)$

$= 2(2x-3)+3$

$= 4x-6+3$

$= \underline{\underline{4x-3}}$

$g(f(x)) = g(2x+3)$

$= 2(2x+3)-3$

$= 4x+6-3$

$= \underline{\underline{4x+3}}$

b) $f(g(x)) \times g(f(x))$

$= (4x-3)(4x+3)$ → 2 squares?

$= 16x^2-9$

$\underline{\underline{\text{minimum} = -9.}}$

4a) Since $-1 < 0.8 < 1$, limit exists.

b) $L = 0.8L+12$

$0.2L = 12$

$L = \dfrac{12}{0.2}$

$L = \dfrac{120}{2}$

$\underline{\underline{L = 60}}$

5. $f(x) = (2x-1)^5$

$f'(x) = 5(2x-1)^4 \times 2$

$= \underline{\underline{10(2x-1)^4}} = 0$ at SPs

$10(2x-1)^4 = 0$ $\qquad f(1/2) = (2(1/2)-1)^5$

$2x-1 = 0$ $\qquad y = 0$

$\underline{x = 1/2}$ $\qquad (1/2, 0)$

	→	1/2	→
$(2x-1)^4$	+	0	+
	/	—	/

raising pt of inflection at $(1/2, 0)$

6a) $\int_0^1 x^3 - 6x^2 + 4x + 1 \; dx$

$\left[\frac{x^4}{4} - 2x^3 + 2x^2 + x\right]_0^1$

$= \left[\frac{1}{4} - 2 + 2 + 1\right] - [0]$

$= \frac{5}{4}$ units^2

b) $\int_1^2 x^3 - 6x^2 + 4x + 1$

$\left[\frac{x^4}{4} - 2x^3 + 2x^2 + x\right]_1^2$

$\left[\frac{16}{4} - 16 + 8 + 2\right] - \frac{5}{4}$

$= -2 - \frac{5}{4}$

$= -\frac{13}{4} = \frac{13}{4}$ units^2

Total area $= \frac{5}{4} + \frac{13}{4}$

$= \frac{18}{4}$

$= \frac{9}{2}$ units^2

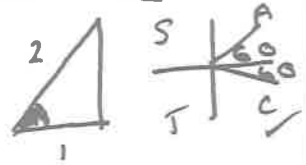
7. $\sin x - \sin 2x = 0$

$\sin x - 2\sin x \cos x = 0$

$\sin x (1 - 2\cos x) = 0$

$\sin x = 0$ $\qquad$ $\cos x = 1/2$

$x = 0, 180, 360$



$x = 60, 300°$

8a) $2x^2 + 4x - 3 = a(x+p)^2 + q$

$= ax^2 + 2apx + ap^2 + q$

$a = 2$ $\qquad$ $2ap = 4$ $\qquad$ $ap^2 + q = -3$

$2(2)p = 4$ $\qquad$ $2(1^2) + q = -3$

$p = 1$ $\qquad$ $2 + q = -3$

$q = -5$

$2(x+1)^2 - 5$

TP : $(-1, -5)$

9a) $\underline{u} \cdot \underline{v} = 1$

$k^3 + 3k^2 - (k+2) = 1$

$k^3 + 3k^2 - k - 2 = 1$

$k^3 + 3k^2 - k - 3 = 0$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -1 & -3 \\ & 0 & -3 & 0 & 3 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$$
$\therefore$ factor

$(k+3)(k^2-1) = 0$

$(k+3)(k-1)(k+1) = 0$

$k > 0 \therefore k = 1$

$\underline{u} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ $\qquad$ $\underline{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

$|\underline{u}| = \sqrt{11}$ $\qquad$ $|\underline{v}| = \sqrt{11}$

$\cos\theta = \dfrac{1 + 3 - 3}{\sqrt{11}\,\sqrt{11}}$

$= \frac{1}{11}$

10. $y = a^x$

$\log_4 y = \log_4 a^x$

$\log_4 y = x \log_4 a$

$m = \frac{3-0}{6-0} = \frac{1}{2}$

$\log_4 a = \frac{1}{2}$

$a = 4^{1/2}$

$a = 2$

$y = 2^x$

Higher 2006 Paper 2

1. (4,6)

$m_{PS} = \frac{6-0}{4-2} = \frac{6}{2} = 3$

If ⊥, $m_1 m_2 = -1$

$3 \times (-1/3) = -1$

$y - 6 = -1/3(x-4)$

$3y - 18 = -x + 4$

$3y = -x + 22$

$y = \frac{-x}{3} + \frac{22}{3}$

b) when $y = 0$,

$0 = -x + 22$

$\underline{\underline{x = 22}}$

Q(22,0)

$PQ = 22 - 2$

$\underline{\underline{= 20}}$

R(24,6)

2. $b^2 - 4ac = 0$ for equal roots

$k^2 - 4(k)(6) = 0$

$k^2 - 24k = 0$

$k(k-24) = 0$

$\underline{k=0}, \underline{k=24}$

Since $k \neq 0$, $\underline{\underline{k = 24}}$

3a) $y = x^2 - 14x + 53$

$\frac{dy}{dx} = 2x - 14$

at $x = 8$

$\frac{dy}{dx} = 2(8) - 14$

$\underline{= 2}$

(8,5)

$m = 2$

$y - 5 = 2(x-8)$

$y - 5 = 2x - 16$

$\underline{\underline{y = 2x + 11}}$

b) let $2x - 11 = -x^2 + 10x - 27$

$x^2 - 8x + 16 = 0$

If tgt, $b^2 - 4ac = 0$

$(-8)^2 - 4(1)(16)$

$= 64 - 64$

$\underline{\underline{= 0 \therefore \text{tangent}}}$

$(x^2 - 8x + 16) = 0$

$(x-4)(x-4) = 0$

$\underline{x = 4}$

$y = 2(4) - 11$

$= 8 - 11$

$\underline{= -3}$

$\underline{\underline{Q = (4,-3)}}$

4. $C(3,4)$

$-k = -2f$

$-k = -2(3)$

$\underline{\underline{k = 6}}$

$x^2 + y^2 - 6x - 8y - 12 = 0$

$r = \sqrt{3^2 + 4^2 - (-12)}$

$= \sqrt{9 + 16 + 12}$

$\underline{\underline{= \sqrt{37}}}$

5) $\frac{dy}{dx} = 4x - 6x^2$ at $(-1, 9)$

$y = \int 4x - 6x^2 \, dx$

$y = [2x^2 - 2x^3 + c]$

$9 = 2(-1)^2 - 2(-1)^3 + c$

$9 = 2 + 2 + c$

$9 = 4 + c$

$\underline{\underline{c = 5}}$

$y = \underline{\underline{2x^2 - 2x^3 + 5}}.$

6a)

$\vec{PQ} = \underline{q} - \underline{p}$

$= \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

$|\vec{PQ}| = \sqrt{4^2+0^2+(-3)^2}$

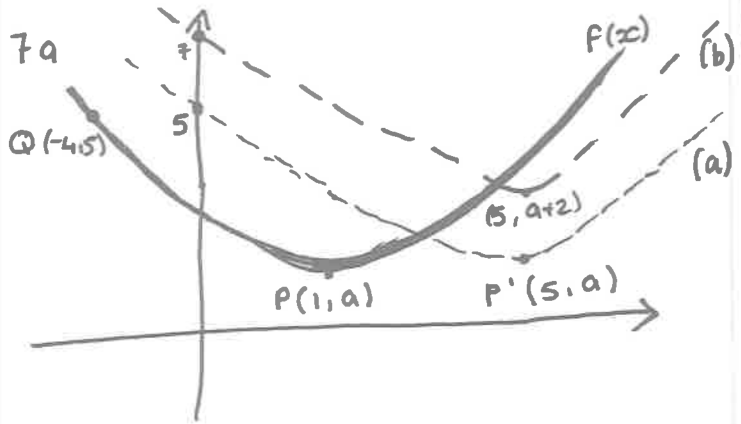
$= \underline{\underline{5}}$

$|\vec{PQ}| = 5$

$\frac{1}{5}|\vec{PQ}| = 1$

$\frac{1}{5}\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 4/5 \\ 0 \\ -3/5 \end{pmatrix}$

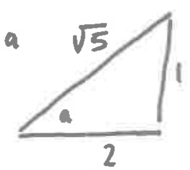
7a



a) $f(x-4)$: $P(5,a)$ ------
$Q(0,5)$

b) $2+f(x-4)$; $P(5,a+2)$ - - - -
$Q(0,7)$

8a



$\sin a = \frac{1}{\sqrt{5}}$ $\cos a = \frac{2}{\sqrt{5}}$

$\sin 2a = 2\sin a \cos a$

$= 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$

$= \underline{\underline{\frac{4}{5}}}$

$\sin 3a = \sin(2a+a)$

$= \sin 2a \cos a + \cos 2a \sin a$

$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2$

$= 1 - \frac{2}{5}$

$= 3/5.$

$\frac{4}{5} \times \frac{2}{\sqrt{5}} + \frac{3}{5} \times \frac{1}{\sqrt{5}}$

$= \frac{8}{5\sqrt{5}} + \frac{3}{5\sqrt{5}}$

$= \underline{\underline{\frac{11}{5\sqrt{5}}}}$

9. $y = x^{-3} - \cos 2x$

$\frac{dy}{dx} = -3x^{-4} - (-\sin 2x) \times 2$

$= \frac{-3}{x^4} + 2\sin 2x$

$= \underline{\underline{2\sin 2x - \frac{3}{x^4}}}$

10a)

$7\sin x - 24\cos x = R\sin x\cos a - R\cos x\sin a$

$R\sin a = 24$

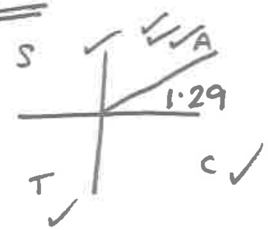
$R\cos a = 7$

$\tan a = \frac{24}{7}$

$a = 1{\cdot}29$ radians

$R = \sqrt{24^2+7^2}$

$= \underline{\underline{25}}$



$y = 25\sin(x - 1{\cdot}29)$

$\frac{dy}{dx} = 25\cos(x - 1{\cdot}29) = 1$

$\cos(x - 1{\cdot}29) = \frac{1}{25}$

$x - 1{\cdot}29 = 1{\cdot}53$

$\underline{\underline{x = 2{\cdot}82}}$ radians

11. $A(t) = A_0 e^{-0.000124t}$

$0{\cdot}88 = 1e^{-0.000124t}$

$\ln 0{\cdot}88 = -0{\cdot}000124t$

$t = \frac{\ln 0{\cdot}88}{-0{\cdot}000124}$

$= 1031$ years

$\therefore$ Claim is true.

12. $PS = 6 - x$

$RS = 12 - \frac{8}{x}$

$A(x) = (6-x)\left(12 - \frac{8}{x}\right)$

$= 72 - \frac{48}{x} - 12x + 8$

$= 80 - 12x - \frac{48}{x}$

$= 80 - 12x - 48x^{-1}$

$A'(x) = -12 + 48x^{-2} = 0$ at max/min

$\frac{48}{x^2} = 12$

$48 = 12x^2$

$48 - 12x^2 = 0$

$12(4 - x^2) = 0$

$12(2-x)(2+x) = 0$

Since $x > 0$, $\underline{x = 2}$

$A(2) = 80 - 12(2) - \frac{48}{2}$

$= 80 - 48$

$= \underline{\underline{32}}$

$\underline{\underline{\text{TP } (2, 32)}}$

$A'(x)$	$\rightarrow$	2	$\rightarrow$
$12(2-x)(2+x)$	$+$	0	$-$
	$/$	$-$	$\backslash$

max at $x = 2$.

Check boundaries $1 < x < 4$

$A(1) = 80 - 12 - 48$

$= \underline{20}$

$A(4) = 80 - 48 - \frac{48}{4}$

$= 80 - 60$

$= 20$

Max at $(2, 32) \rightarrow \underline{\underline{32 \text{ units}^2}}$

min at $(1, 20)$ and $(4, 20)$

$\Rightarrow \underline{\underline{20 \text{ units}^2}}$