2011 Paper 1

1. $\begin{pmatrix}4\\10\\-14\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \begin{pmatrix}-2\\1\\0\end{pmatrix} = \begin{pmatrix}5\\9\\-13\end{pmatrix}$ (C)

2. $3y + 2x = 6$
$3y = -2x + 6$
$y = \left(\frac{-2}{3}\right)x + 2$ (B)

3. Slide left 2, down 1 (D)

4. $\frac{dy}{dx} = 3x^2 - 2$
at $x = 2$
$\frac{dy}{dx} = 3(2^2) - 2$
$= 12 - 2$
$= 10$ (D)

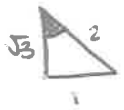
5. $x^2 - 8x + 7 = (x-4)^2 - 9$
$q = -9$ (A)

6. $\therefore m = \frac{1}{2}$ $(a, b) = (2, -3)$
$y + 3 = \frac{1}{2}(x - 2)$ (C)

7.
	1	−1	1	3
1		0	1	0	1
	1	0	1	4

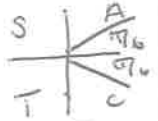
(D)

8. $\tan 30 = m$. (A)



9. $\sqrt{23}$ - not rational (2 false)
> 0 - real (1 true) (B)

10. $2\cos x = \sqrt{3}$
$\cos x = \sqrt{3}/2$
$x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$.



(D)

11. $\int 4x^{1/2} + x^{-3}\,dx$
$= \frac{4x^{3/2}}{3/2} + \frac{x^{-2}}{-2} + C$
$= \frac{8x^{3/2}}{3} - 2x^{-2} + C$ (D)

12. $\sin(p+q) = \sin p\cos q + \sin q\cos p$
$= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3} + \frac{2}{3} \times \frac{1}{\sqrt{5}}$
$= \frac{2\sqrt{5}}{3\sqrt{5}} + \frac{2}{3\sqrt{5}}$ (C)

13. $f(x) = 4\sin 3x$
$f'(x) = 12\cos 3x$
$= 12(3)$
$= 36$ (D)

14. $\underline{p} \cdot \underline{q} = |\underline{p}||\underline{q}|\cos\theta$
$= (3)(3)\cos 60$
$= 9\left(\frac{1}{2}\right)$
$= \frac{9}{2}$

all angles 60°.

15. $\vec{ST} = t - s$
$= \begin{pmatrix}-12\\-9\\15\end{pmatrix}$
$= 3\begin{pmatrix}-4\\-3\\5\end{pmatrix}$

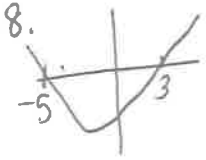
$\vec{TU} = \begin{pmatrix}-8\\-6\\10\end{pmatrix}$
$= 2\begin{pmatrix}-4\\-3\\5\end{pmatrix}$

(B)

16. $\int \frac{1}{3}x^{-4}\,dx$
$= \frac{1x^{-3}}{3 \times -3} + C$
$= \frac{-1}{9x^3} + C$ (A)

17. $y = kx(x+1)(x-2)$ from roots ($x = 0$ too).
at $(1, 2)$
$2 = k(2)(-1)$
$2 = k(-2)$
$k = -1$
$y = -x(x+1)(x-2)$ (A)

18.



$x < -5$
$x > 3$ (C)

2011 Paper 1

19. $\log_3 y = x$

$y = 3^x$.

$y = 3^x \Rightarrow 3^1 = 3 \therefore (1,3)$

$\log_3 y = x \therefore (3,1)$ ①

20. $(x-2)$ cannot be negative

$\therefore x \geq 2$

as $\sin^2 x$ is always positive ©

21 BD

$m_{BD} = \frac{y_B - y_D}{x_B - x_D}$

$= \frac{12-(-3)}{7-2}$

$= \frac{15}{5}$

$= 3$

at (7,12)

$y - 12 = 3(x-7)$

$y - 12 = 3x - 21$

$y = 3x - 9$

b) $x + 3y = 23$, $y = 3x - 9$

$x + 3(3x-9) = 23$

$x + 9x - 27 = 23$

$10x = 50$

$x = 5$

$y = 3(5) - 9$ E(5,6)

$= 15 - 9$

$y = 6$

c) $m_{AB} = \frac{y_A - y_B}{x_A - x_B}$

$= \frac{8-12}{-1-7}$

$= \frac{-4}{-8}$

$= \frac{1}{2}$

$M_{AB} = (3,10)$

$y - 10 = -2(x-3)$

$y - 10 = -2x + 6$

$y = -2x + 16$

if $b_1 m_1 m_2 = -1$

$m_{bisector} = -2$.

c) $y = -2x + 16$

$y = -2(5) + 16$

$= -10 + 16$

$y = 6$

$\therefore (5,6)$ satisfies equation

$\therefore$ lies on line.

22. $f(x) = (x-2)(x^2+1)$

on x axis, $y = 0$.

$x = 2$ $x^2 = -1$ no solution

on y axis, $x = 0$.

$y = (-2)(1)$

$y = -2$

i) $f(x) = x^3 + x - 2x^2 - 2$

$f'(x) = 3x^2 - 4x + 1 = 0$ at SPs

$(3x - 1)(x - 1) = 0$

$x = 1/3$ $x = 1$

$f(x) = x^3 - 2x^2 + x - 2$

$f(1) = 1^3 - 2(1^2) + 1 - 2$ (1, -2)

$= 1 - 2 + 1 - 2$

$= -2$

$f(1/3) = (1/3)^3 - 2(1/3^2) + (1/3) - 2$

$= 1/27 - 2/9 + 1/3 - 2$

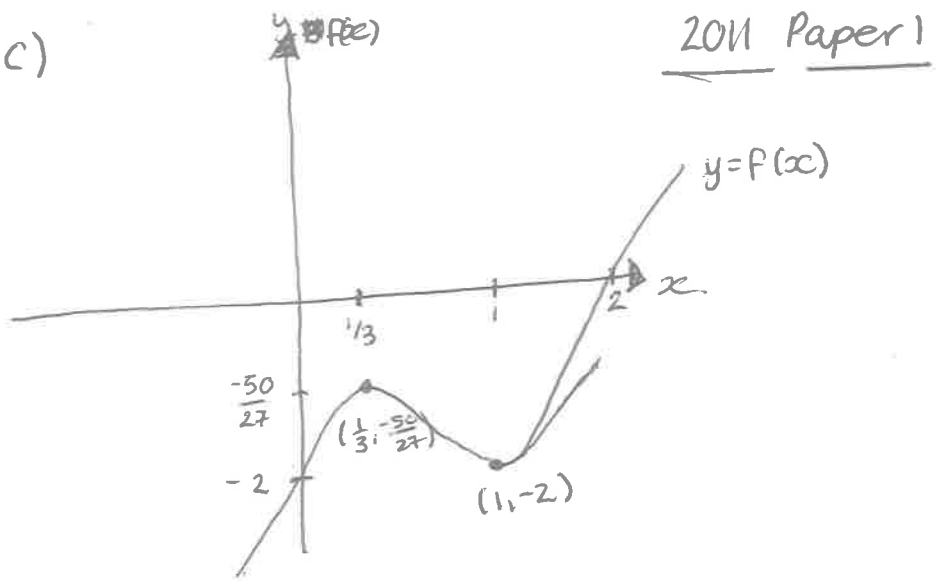
$= 1/27 - 6/27 + 9/27 - 54/27$

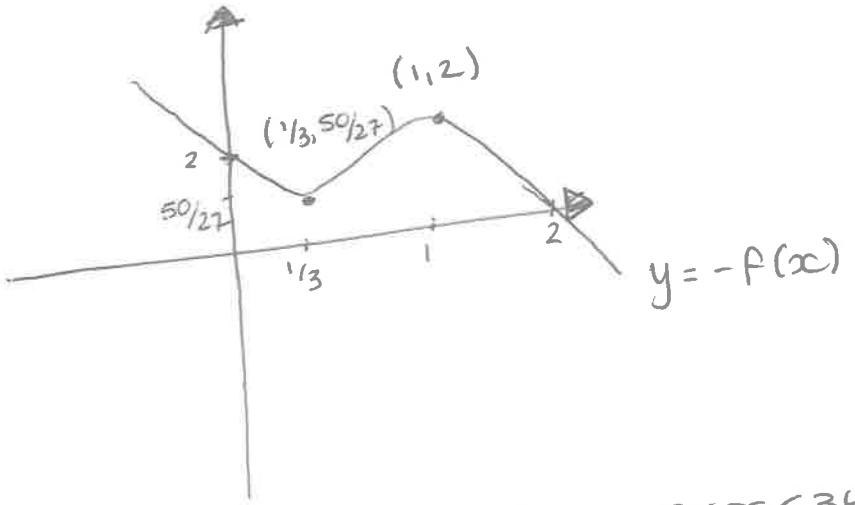
$= 10/27 - 60/27$

$= -50/27$ $(1/3, -50/27)$

$f'(x)$	→	$1/3$	→	1	→
$(3x-1)$	−	0	+	0	+
$(x-1)$	−		−		+
	\ max at $(1/3, -50/27)$		/	min at $(1,-2)$.	

c)





2011 Paper 1

23a) $\cos 2x - 3\cos x + 1 = 0$ $\quad 0 < x < 360$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

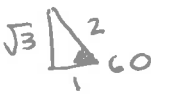
$2\cos x - 1 = 0$ $\qquad \cos x = 1$

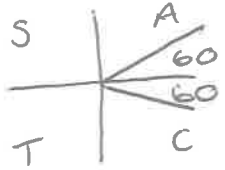
$\cos x = \frac{1}{2}$ $\qquad x = 0°, 360°$

$\cos^{-1}(1/2) = 60°$

$x = 60°, 300°$

$x° = 0°, 60°, 300°, 360°$





c) $\cos 4x - 3\cos 2x + 2 = 0$

$(2\cos 2x - 1)(\cos 2x - 1)$

$2x = 0, 60, 300, 360°, 420°, 660°, 720°$

$x = 0, 30, 150, 180°, 210°, 330°, 360°.$

2011 Paper 2

1a) B(4,4,0)

b) $\vec{DB} = \underline{b} - \underline{d}$

$= \begin{pmatrix}4\\4\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix}$

$= \begin{pmatrix}2\\2\\-6\end{pmatrix}$

$\vec{DM} = \underline{m} - \underline{d}$

$= \begin{pmatrix}2\\2\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix}$

$= \begin{pmatrix}0\\0\\-6\end{pmatrix}$

c) $|\vec{DB}| = \sqrt{2^2+2^2+(-6)^2}$

$= \sqrt{4+4+36}$

$= \sqrt{44}$

$|\vec{DM}| = \sqrt{36}$

$= 6$

$\vec{DB} \cdot \vec{DM} = 0 + 0 + 36$

$\cos BDM = \dfrac{\vec{DB} \cdot \vec{DM}}{|\vec{DB}||\vec{DM}|}$

$= \dfrac{36}{6\sqrt{44}} = \dfrac{6}{\sqrt{44}} = \dfrac{6}{2\sqrt{11}} = \dfrac{3}{\sqrt{11}}$

$BDM = \cos^{-1}\left(\dfrac{3}{\sqrt{11}}\right)$

$= 25.239\ldots$

$= \underline{25.2^\circ}$

2. $f(x) = x^3 - 1$ $\quad g(x) = 3x + 1$

$g(f(x)) = g(x^3-1)$ $\quad g(x^3-1) = 3(x^3-1) + 1$

$= \underline{3x^3 - 3 + 1}$

$= \underline{3x^3 - 2}$

b) $g(f(x)) + xh(x)$

$= 3x^3 - 2 + x(4x - 5)$

$= 3x^3 - 2 + 4x^2 - 5x$

$= 3x^3 + 4x^2 - 5x - 2$

c) $x - 1 = 0 \therefore x = 1$

1	3	4	−5	−2
	0	3	7	2
	3	7	2	0 ∴ factor

$(x-1)(3x^2 + 7x + 2)$

$(x-1)(3x + 1)(x + 2) = 0$

d) $\underline{x = 1}$ $\quad \underline{x = -1/3}$ $\quad \underline{x = -2}$

3a) $u_{n+1} = -\frac{1}{2}u_n$

$u_1 = -\frac{1}{2}(-16) = 8$

$u_2 = -\frac{1}{2}(8) = -4$

b) $V_{n+1} = pV_n + q$

$7 = 5p + q$

$-\ 5 = 4p + q$

$2 = p$

$4(2) + q = 5$

$8 + q = 5$

$\underline{q = -3}$

$V_1 = 4$

$V_2 = 5$

$V_3 = 7$

$V_{n+1} = 2V_n - 3$.

2011 Paper 2

c) $U_{n+1} = -\frac{1}{2}U_n$ has a limit as $-1 < -\frac{1}{2} < 1$.

$L = -\frac{1}{2}L$

$\frac{3}{2}L = 0$

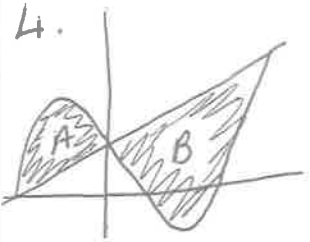
$L = 0$

Since $2 > 0$

$V_{n+1} = 2V_n - 3$

does not have a limit

4.



$A = x^3 - x^2 - 4x + 4 - (2x + 4)$

$= x^3 - x^2 - 4x + 4 - 2x - 4$

$= x^3 - x^2 - 6x$

$B = 2x + 4 - (x^3 - x^2 - 4x + 4)$

$= 2x + 4 - x^3 + x^2 + 4x - 4$

$= 6x + x^2 - x^3$.

$$\int_0^3 B\,dx + \int_{-2}^0 A\,dx.$$

$$\int_0^3 (6x + x^2 - x^3)\,dx + \int_{-2}^0 (x^3 - x^2 - 6x)\,dx$$

$$= \left[3x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^3 - \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right]_{-2}^0$$

$$= \left[3(9) + \frac{27}{3} - \frac{81}{4}\right] - [0] - \left[[0] - \left[\frac{16}{4} + \frac{8}{3} - 12\right]\right]$$

$$= \left[27 + 9 - \frac{81}{4}\right] - \left[4 + \frac{8}{3} - 12\right]$$

$$= \left[36 - \frac{81}{4}\right] - \left[-8 + \frac{8}{3}\right]$$

$$= \left[\frac{144}{4} - \frac{81}{4}\right] - \left[\frac{-24}{3} + \frac{8}{3}\right]$$

$$= \frac{63}{4} + \frac{16}{3}$$

$$= \frac{189}{12} + \frac{64}{12}$$

$$= \frac{253}{12}$$

$= 21\frac{1}{12}$ units^2.

5. $y = kx^n$

(0, 5) (4, 7)

$m = \frac{7-5}{4-0}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$y = kx^n$

$\log y = \log kx^n$

$\log y = \log k + \log x^n$

$\log y = n \log x + \log k$.

$Y = mX + c$

$y = \frac{1}{2}x + 5$.

$\log_2 k = 5$

$k = 2^5$

$k = 32$

$y = 32x^{\frac{1}{2}}$

2011 Paper 2

RADIANS

6a) $3\sin x - 5\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$.

$R\cos\alpha = 3$

$R\sin\alpha = -5$.

$\tan\alpha = \frac{-5}{3}$

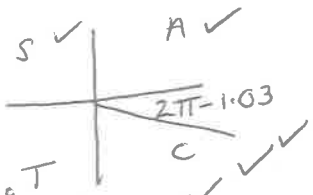
$R^2 = \sqrt{(3^2)+(-5)^2}$

$= \sqrt{9+25}$

$= \underline{\sqrt{34}}$

$\tan^{-1}(\frac{5}{3}) = 1.03$ rads

$\alpha = 5.3$ rads.



$3\sin x - 5\cos x = \sqrt{34}\sin(x+5.3)^\circ$ ✓

b) $\int_0^t \sqrt{34}\cos(x+5.3)\,dx = 3$.

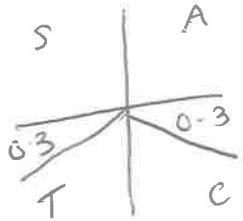
$\left[\sqrt{34}\sin(x+5.3)\right]_0^t = 3$

$\sqrt{34}\sin(t+5.3) - \sqrt{34}\sin(5.3) = 3$.

$\sqrt{34}\sin(t+5.3) + 4.9 = 3$.

$\sqrt{34}\sin(t+5.3) = -1.9$

$\sin(t+5.3) = \frac{-1.9}{\sqrt{34}}$



$\sin^{-1}(\frac{1.9}{\sqrt{34}}) = 0.3$ rads

$t+5.3 = 3.4, 6.0$

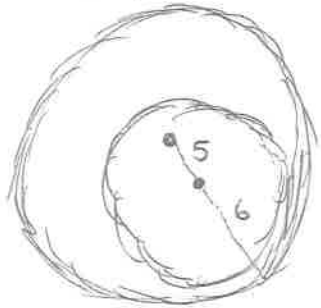
$t = 0.7, 4.4$ rads

since $0<t<2$, $\underline{t=0.7}$

7. C_1: centre $(-1,1)$ radius $=11$.

C_2: centre $(2,-3)$ radius $= \sqrt{2^2+(-3)^2-p}$

$= \sqrt{13-p}$



$d_{C_1C_2} = \sqrt{(2-(-1))^2+(-3-1)^2}$

$= \sqrt{3^2+(-4)^2}$

$= \sqrt{9+16}$

$= \underline{5}$

if circles have no point of contact, the radius of the inner circle must be less than 6.

$\sqrt{13-p} < 6$

$13-p < 36$

$-p < 23$

$\underline{p > -23}$