NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2007/2008
Paper 1
Assessing Units 1, 2 & 3

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2007/2008 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

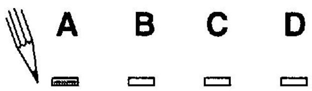
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

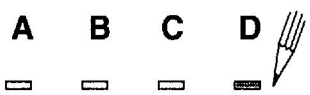
A 2
B 27
C $1{\cdot}5$
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$ $\cos ax$	$a\cos ax$ $-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$ $\cos ax$	$-\frac{1}{a}\cos ax + C$ $\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted

1. A line has as its equation $3y = x + 6$.
Any line parallel to this line will have as its gradient

A -3
B 1
C $-\frac{1}{3}$
D $\frac{1}{3}$

2. If $f(x) = \frac{1}{x^3}$ and $x \neq 0$, then $f'(x)$ is

A $\frac{1}{3x^2}$

B $-\frac{3}{x^4}$

C $-\frac{3}{x^2}$

D $-\frac{1}{2x^2}$

3. The remainder when $2x^3 + x^2 - 1$ is divided by $x - 2$ is

A 9
B 5
C 19
D -13

4. Which of the following is/are true of the circle with equation $x^2 + y^2 - 36 = 0$?

1 It passes through the origin.
2 It has a radius of 6.
3 It has the origin as its centre.

A 1 only
B 2 only
C 2 and 3 only
D some other combination of responses

5. Given that $\cos x^\circ = \frac{1}{\sqrt{3}}$ and $0 < x < 90$,
then the exact value of $\cos 2x^\circ$ will be

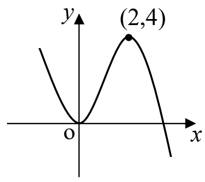
A $\frac{2}{\sqrt{3}}$

B $-\frac{1}{3}$

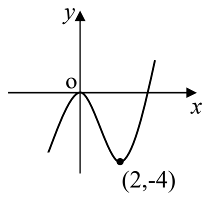
C $\frac{1}{3}$

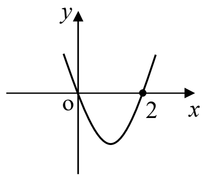
D $\frac{1}{2\sqrt{3}}$

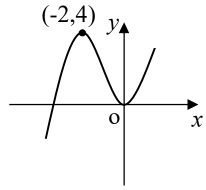
6. Part of the graph of $y = f(x)$ is shown below.

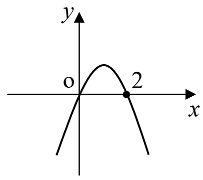


The graph of $y = f'(x)$ could be represented by

A


B


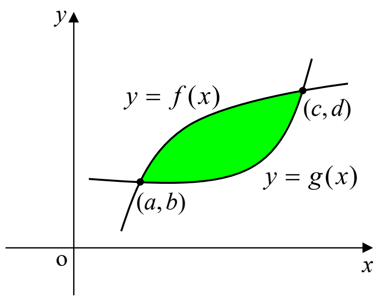
C


D


7. Which one of the following points lies on the graph of $y = \log_3 x$?

A (9,2)
B (3,27)
C (2,9)
D (0,0)

8.



The shaded area above is given by

A $\int_b^d \left(f(x) - g(x)\right) dx$

B $\int_a^c \left(f(x) + g(x)\right) dx$

C $\int_a^c \left(f(x) - g(x)\right) dx$

D $\int_a^d \left(f(x) - g(x)\right) dx$

9. Two functions, defined on suitable domains, are given as $f(x) = 3x^2 - 2$ and $g(x) = 1 - x$. The value of $f(g(2))$ is

A -9
B -5
C -1
D 1

10. The value of $\cos\frac{5\pi}{6}$ is

A $-\frac{1}{2}$
B $-\frac{\sqrt{3}}{2}$
C $\frac{\sqrt{3}}{2}$
D $\frac{1}{2}$

11. Given that $\boldsymbol{v} = \begin{pmatrix} \sqrt{2} \\ 2 \\ \sqrt{3} \end{pmatrix}$, then $|\boldsymbol{v}|$ is

A $2 + \sqrt{5}$
B 3
C 9
D $\sqrt{7}$

12. A circle has as its equation $x^2 + y^2 + 4x - 2y - 4 = 0$. Which of the following correctly states the coordinates of its centre and the value of its radius?

A $(-2, 1)$, $r = 1$
B $(2, -1)$, $r = 3$
C $(-2, 1)$, $r = 3$
D $(2, -1)$, $r = 1$

13. $\int_0^{\frac{\pi}{4}} 4(\cos 2x)\, dx$ is equal to

A 0
B 4
C -2
D 2

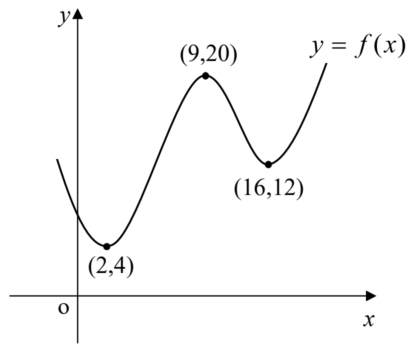
14. A recurrence relation is defined by $U_{n+1} = 0 \cdot 4U_n - 24$.
The limit of this sequence is

A -40
B -24
C $0 \cdot 03$
D 50

15. If x and y are integers the value of $(x + y)^2 - (x - y)^2$ is always

A negative
B positive
C a perfect square
D a multiple of 4

16.



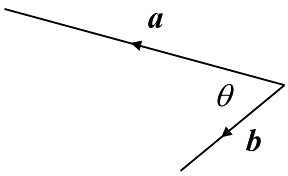
The diagram shows part of the graph of $y = f(x)$.

Which of the following is/are true for the function above?

1 $f'(0) < 0$
2 $f'(6) < 0$
3 $f'(9) = 0$
4 $f'(12) > 0$

A 2 and 3 only
B 3 only
C 1 and 3 only
D 1, 2, 3 and 4

17. Consider the diagram and information below.

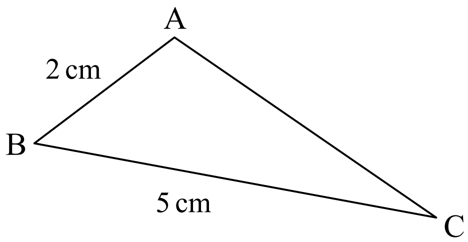


$\cos\theta = \frac{1}{2}$

If the magnitude of vector **a** is 2 and the magnitude of vector **b** is 1 then the value of $\mathbf{a}.(\mathbf{a}+\mathbf{b})$ is

A 6
B $\sqrt{5}$
C 5
D 3

18.



If $\tan \text{ABC} = \frac{3}{4}$ then the area of triangle ABC in square centimetres is

A 5
B 4
C $\frac{15}{4}$
D 3

19. The quadratic equation $4kx^2 - 8x + k = 0$ has equal roots.
The value of k, where $k > 0$ is

A 4
B 2
C 0
D -2

20. $f(x) = ax^2 - 2x - 5$ has a stationary value when $x = 3$.
The value of a is

A $\frac{1}{3}$
B $-\frac{1}{3}$
C $\frac{7}{6}$
D $\frac{11}{9}$

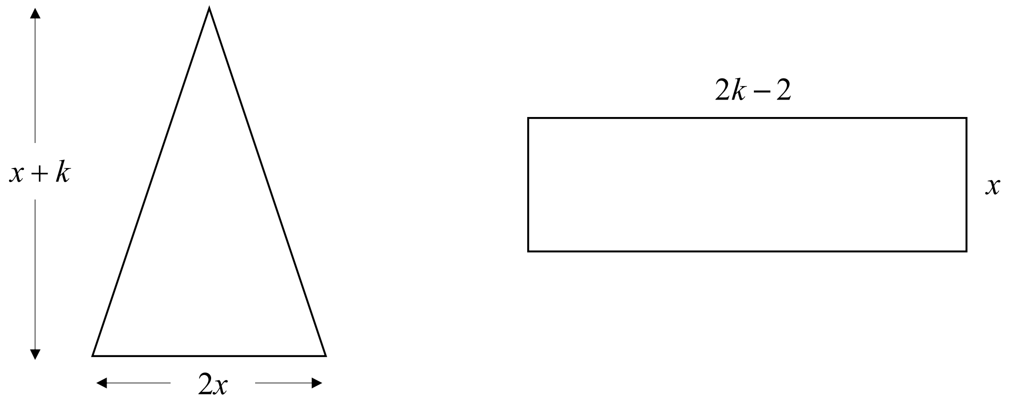
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. Consider the isosceles triangle and the rectangle below.

The triangle has a base measuring $2x$ and a vertical height of $x + k$.
The rectangle has dimensions $2k - 2$ by x as shown.
All dimensions are in centimetres.



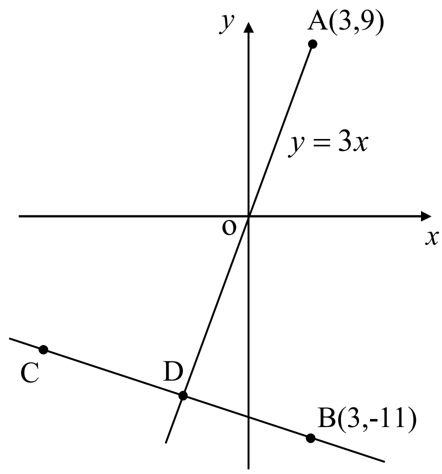
(a) Given that the **area of the rectangle** is 4cm^2 **more than** the area of the triangle, **show clearly** that the following equation can be formed.

$$x^2 + (2 - k)x + 4 = 0$$ **3**

(b) Hence find k, given that the equation $x^2 + (2 - k)x + 4 = 0$ has equal roots and $k > 0$. **3**

(c) Find x when k takes this value and calculate the area of each shape. **3**

22. In the diagram A has coordinates (3,9) and the point B has coordinates (3,-11) as shown.
A lies on the line with equation $y = 3x$.



(a) If line BC is perpendicular to the line AD, establish the equation of BC. **2**

(b) Hence find the coordinates of D. **3**

(c) If D is the mid-point of BC, write down the coordinates of C. **1**

(d) Find the equation of the circle passing through the points A, D and C. **4**

23. A function is defined on a suitable domain as $f(x) = \frac{1}{3}x^3 - 4x^2 + x$.

(a) Show that its derivative can be expressed in the form

$$f'(x) = (x + p)^2 + q, \text{ and state the values of } p \text{ and } q.$$ **4**

(b) Hence state the minimum rate of change of this function and the corresponding value of x. **2**

24. Find the solution(s) of the equation $2\cos^2 a = \cos a + 1$ for $0 \leq a \leq \pi$. **5**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]

Mathematics

Higher Prelim Examination 2007/2008

Assessing Units 1, 2 & 3

Paper 2

NATIONAL
QUALIFICATIONS

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}\,.\,\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

<u>or</u>

$\boldsymbol{a}\,.\,\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

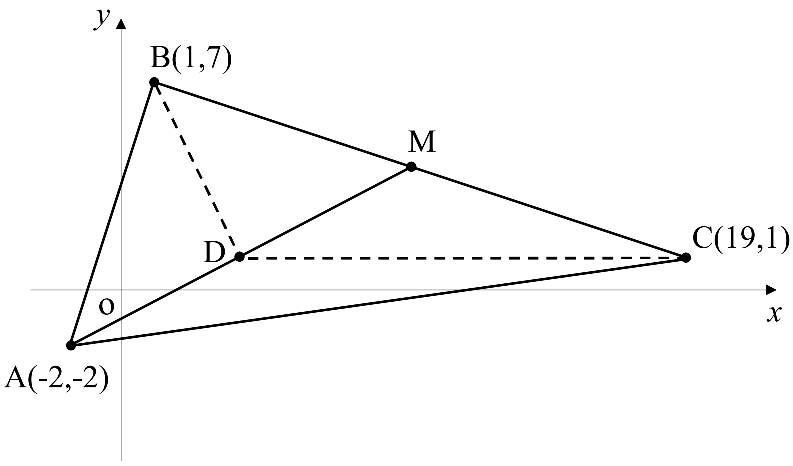
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted

1. Triangle ABC has vertices (-2,-2), (1,7) and (19,1) as shown.

M is the mid-point of side BC.

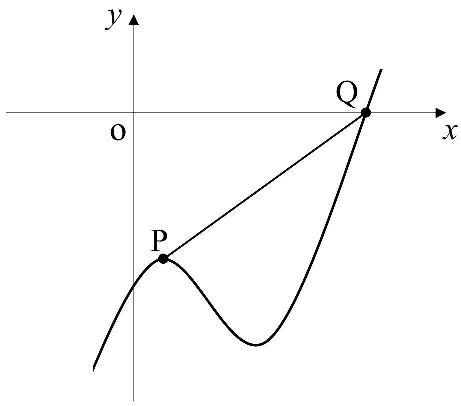


(a) Establish the equation of the median AM. **3**

(b) The **horizontal** line through C intersects AM at D.
Find the coordinates of D. **3**

(c) Hence show clearly that BD is perpendicular to AM. **3**

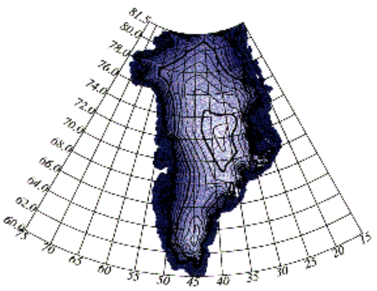
2. Part of the graph of the curve with equation $y = x^3 - \frac{15}{2}x^2 + 12x - 18$ is shown below.
The graph is not drawn to scale.

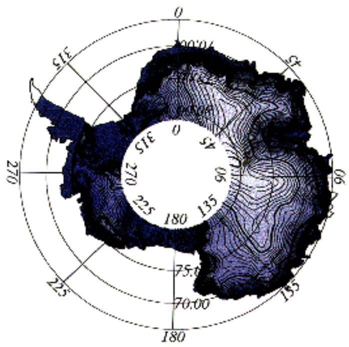


(a) Find the coordinates of the stationary point P. **4**

(b) Find the coordinates of Q. **3**

3. A scientist is running a computer simulation to represent the possible shrinkage of a small polar ice sheet due to global warming.





He discovers that for this particular simulation the ice sheet is losing 4% of its mass every **2 months**.

(a) Calculate the mass of ice remaining after **10 months** if the initial mass of the simulated ice sheet is 40 gigatonnes (approximately 10 cubic miles of ice). Give your answer correct to 3 significant figures. **3**

(b) For the remaining 2 months of the year (the coldest period) there is no mass loss. During this period the ice sheet **gains** 3·8 gigatonnes of mass due to significant snowfall and the partial freezing of the surrounding sea water.

This yearly cycle is then repeated.

By considering an appropriate recurrence relation, calculate the mass of ice remaining after a 3 year period. **3**

(c) The scientist knows that **over the long term** the mass of the ice sheet will always lie between an **upper and lower limit**.

Calculate these two limits.

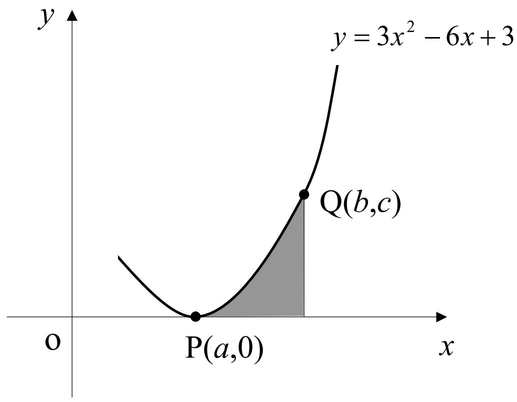
Your answer must be accompanied by appropriate working. **3**

4. Two functions are defined on a suitable domains as $f(x) = x^2 + a$ and $g(x) = x + 1$, where a is a constant.

(a) Find the value of a given that $f\big(g(-2)\big) = -1$ **2**

(b) Hence solve the equation $f\big(f(x)\big) = 2$ **5**

5. The diagram below, which is not drawn to scale , shows part of the graph of the curve with equation $y = 3x^2 - 6x + 3$. The points P(a,0) and Q(b,c) lie on this curve as shown.



(a) Establish the value of a. **2**

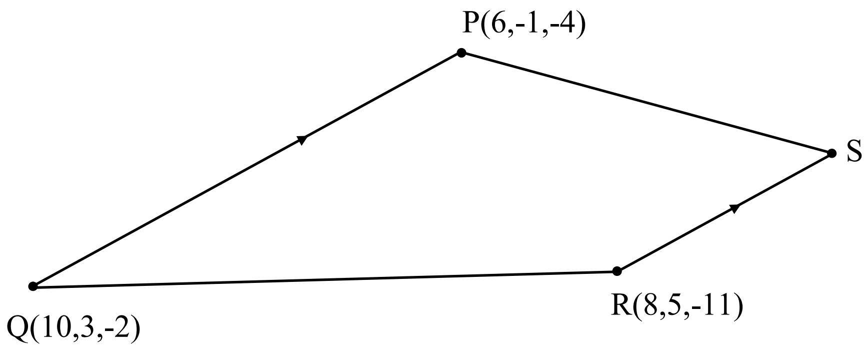
(b) The shaded area (A) can be represented by the integral

$$A = \int_a^b (3x^2 - 6x + 3) \; dx$$

If the shaded area is exactly 1 square unit, find the value of b. **5**

6. Solve $2^{-0\cdot04t} = 0\cdot2$, for t, giving your answer correct to **2 significant figures**. **4**

7. Three vertices of the quadrilateral PQRS are P(6,-1,-4), Q(10 ,3,-2) and R(8,5,-11).

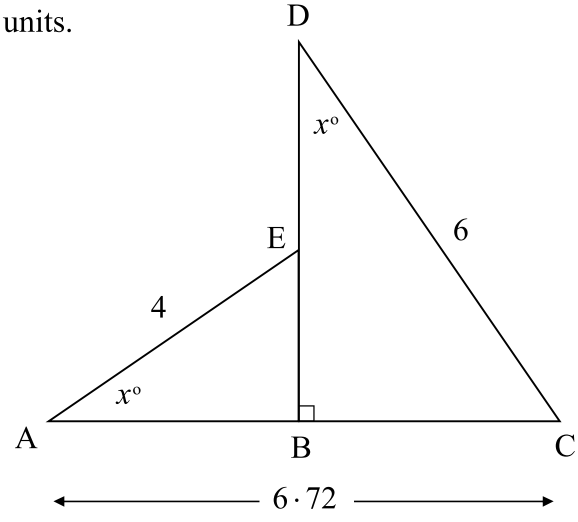


(a) Given that $\overrightarrow{QP} = 2\overrightarrow{RS}$, establish the coordinates of S. **3**

(b) Hence show that angle PSR is a right angle. **3**

8. The diagram below shows two right-angled triangles with $\angle EAB = \angle CDB = x°$.
Side AE = 4 units and side DC = 6 units.

The length from A to C is $6 \cdot 72$ units.



(a) **By considering expressions for the lengths of AB and BC** show clearly that the following equation can be formed

$$4\cos x° + 6\sin x° = 6 \cdot 72 .$$ **1**

(b) Hence solve the equation $4\cos x° + 6\sin x° = 6 \cdot 72$ for x where $0 < x < 45$.
Give your answer correct to the nearest degree. **5**

9. An amateur rocketeer has built a rocket which he hopes will reach a height of at least 4000 feet when using his own home made liquid fuel.

He has modelled the height reached to the mass of fuel used by the formula

$$H(m) = 4m - \frac{m^2}{1200},$$

where H is the height reached in **feet** and m is the mass of fuel used in **millilitres** (ml).

(a) Find the mass of fuel he should use to propel his rocket to its **maximum** height. **4**

(b) What is the predicted maximum height for this rocket when m takes this value? **1**

[END OF QUESTION PAPER]

Mathematics

Higher Prelim Examination 2007/2008

NATIONAL QUALIFICATIONS

Paper 1 - Section A - Answer Sheet

NAME :

CLASS :

TEACHER :

You should use an HB pencil.
Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example

A	B	C	D
▭	▬	▭	▭

	A	B	C	D
1	▭	▭	▭	▭
2	▭	▭	▭	▭
3	▭	▭	▭	▭
4	▭	▭	▭	▭
5	▭	▭	▭	▭
6	▭	▭	▭	▭
7	▭	▭	▭	▭
8	▭	▭	▭	▭
9	▭	▭	▭	▭
10	▭	▭	▭	▭
11	▭	▭	▭	▭
12	▭	▭	▭	▭
13	▭	▭	▭	▭
14	▭	▭	▭	▭
15	▭	▭	▭	▭
16	▭	▭	▭	▭
17	▭	▭	▭	▭
18	▭	▭	▭	▭
19	▭	▭	▭	▭
20	▭	▭	▭	▭

Section A	/ 40	
Section B	/ 30	
Total (P1)	/ 70	
Total (P2)	/ 60	
Overall Total	/ 130	%

Please make sure you have filled in all your details above before handing in this answer sheet.

Higher Grade - Paper 1 2007/2008 ANSWERS - Section A

1	D
2	B
3	C
4	C
5	B
6	D
7	A
8	C
9	D
10	B
11	B
12	C
13	D
14	A
15	D
16	C
17	C
18	D
19	B
20	A

	A	B	C	D
1	□	□	□	■
2	□	■	□	□
3	□	□	■	□
4	□	□	■	□
5	□	■	□	□
6	□	□	□	■
7	■	□	□	□
8	□	□	■	□
9	□	□	□	■
10	□	■	□	□
11	□	■	□	□
12	□	□	■	□
13	□	□	□	■
14	■	□	□	□
15	□	□	□	■
16	□	□	■	□
17	□	□	■	□
18	□	□	□	■
19	□	■	□	□
20	■	□	□	□

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	**ans: proof** **(3 marks)**	
	•[1] finds expressions for 2 areas	•[1] $A_{rect} = x(2k-2)$; $A_{tri} = x(x+k)$
	•[2] adds 4 to area of triangle and equates	•[2] $x(x+k)+4 = x(2k-2)$
	•[3] reorganises to given form	•[3] $x^2 + (2-k)x + 4 = 0$
(b)	**ans: $k = 6$** **(3 marks)**	
	•[1] knows condition for equal roots	•[1] $b^2 - 4ac = 0$ [stated or implied]
	•[2] substitutes values	•[2] $(2-k)^2 - 4\times1\times4 = 0$
	•[3] solves and discards	•[3] $(k+2)(k-6) = 0; k = -2$ or 6; $k = 6$
(c)	**ans: $x = 2$; 20cm^2; 16cm^2** **(3 marks)**	
	•[1] substitutes value of k to form quadratic	•[1] $x^2 - 4x + 4 = 0$
	•[2] solves to x	•[2] $(x-2)^2 = 0; x = 2$
	•[3] finds areas	•[3] $A_{rect} = 20cm^2; A_{tri} = 16cm^2$
22(a)	**ans: $3y + x = -30$** **(2 marks)**	
	•[1] identifies required gradient	•[1] $m_{CB} = -\frac{1}{3}$
	•[2] substitutes into general equation	•[2] $y + 11 = -\frac{1}{3}(x-3)$ [or equivalent]
(b)	**ans: D(-3,-9)** **(3 marks)**	
	•[1] knows to use systems of equations	•[1] evidence
	•[2] finds value for x	•[2] $x = -3$
	•[3] finds value for y and states coordinates	•[3] $y = -9$; (-3,-9)
(c)	**ans: C(-9,-7)** **(1 mark)**	
	•[1] states coordinates of C	•[1] C(-9,-7)
(d)	**ans: $(x+3)^2 + (y-1)^2 = 100$** **(4 marks)**	
	•[1] identifies diameter	•[1] AC is diameter [$\angle$ADC is right-angled]
	•[2] finds centre	•[2] midpoint of AC is (-3,1)
	•[3] finds radius or r^2	•[3] $r = 10$ or $r^2 = 100$
	•[4] subs into general equation	•[4] $(x+3)^2 + (y-1)^2 = 100$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	**ans: $(x-4)^2-15$; p = -4, q = -15 (4 marks)**	
	•[1] finds derivative	•[1] $f(x)=x^2-8x+1$
	•[2] starts to complete square	•[2] $(x-4)^2$ ……..
	•[3] completes	•[3] ……… -15
	•[4] states values of p and q	•[4] p = -4, q = -15
(b)	**ans: -15 when x = 4 (2 marks)**	
	•[1] states minimum rate of change	•[1] rate of change is -15
	•[2] states value of x	•[2] $x = 4$
24	**ans: $\frac{2\pi}{3}$, 0 (5 marks)**	
	•[1] collects terms to LHS and equates to 0	•[1] $2\cos^2 a-\cos a-1=0$
	•[2] factorises quadratic	•[2] $(2\cos a+1)(\cos a-1)=0$
	•[3] finds values for $\cos a$	•[3] $\cos a=-\frac{1}{2}$ or $\cos a=1$
	•[4] finds one value for a	•[4] $\frac{2\pi}{3}$
	•[5] finds second value for a	•[5] 0
		Total: 30 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $2y - x = -2$ (3 marks)**	
	•1 finds midpoint of BC	•1 midpoint BC: (10,4)
	•2 establishes gradient of AM	•2 $m_{AM} = \frac{4+2}{10+2} = \frac{1}{2}$
	•3 substitutes in general equation	•3 $y - 4 = \frac{1}{2}(x - 10)$
(b)	**ans: D(4,1) (3 marks)**	
	•1 realising $y = 1$	•1 $y = 1$
	•2 substitutes into equation	•2 $2(1) - x = -2$; $x = 4$
	•3 states coordinates of D	•3 D(4,1)
(c)	**ans: proof (3 marks)**	
	•1 finds gradient of BD	•1 $m_{BD} = -2$
	•2 knows condition for perp. lines	•2 $m_1 \times m_2 = -1$ [stated or implied]
	•3 makes statement re perpendicular	•3 $\frac{1}{2} \times -2 = -1$ so AM and BD are perp.
2(a)	**ans: P(1,$-\frac{25}{2}$) (4 marks)**	
	•1 knows to take derivative and equate to 0	•1 $\frac{dy}{dx} = 0$
	•2 takes derivative	•2 $3x^2 - 15x + 12 = 0$
	•3 solves to find x - coordinate	•3 $x = 1$ [or 4]
	•4 substitutes to find y - coordinate	•4 $y = 1^3 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$
(b)	**ans: Q(6,0) (3 marks)**	
	•1 knows to make $y = 0$	•1 $y = 0$
	•2 uses synthetic division to find x	•2 $\begin{array}{c\|cccc} 6 & 1 & -\frac{15}{2} & 12 & -18 \\ & & 6 & -9 & 18 \\ \hline & 1 & -\frac{3}{2} & 3 & 0 \end{array}$
	•3 states coordinates of Q	•3 Q(6,0)

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans: 32·6 gigatonnes (3 marks)**	
	•¹ correct multiplier •² completes calculation •³ calculation and correct rounding	•¹ 0·96 •² $0{\cdot}96^5 \times 40$ •³ 32·6 gigatonnes
(b)	**ans: 31 gigatonnes (3 marks)**	
	•¹ sets up recurrence relation •² knows to calculate 3 figures •³ final answer	•¹ $U_{n+1} = 0\cdot96^5 U_n + 3\cdot8$ •² 1^{st} year: 36·4; 2^{nd} year: 33·4795 •³ 3^{rd} year: 31 gigatonnes
(c)	**ans: upper 20·6; lower 16·8 (3 marks)**	
	•¹ knows limit exists •² finds upper limit •³ finds lower limit	•¹ limit exists since $-1 < 0{\cdot}96^5 < 1$ •² $L = \dfrac{3\cdot8}{1-(0\cdot96)^5} = 20{\cdot}6$ •³ 20·6 – 3·8 = 16·8
4(a)	**ans: a = -2 (2 marks)**	
	•¹ finds expression for $f(g(-2))$ •² equates to -1 and solves for a	•¹ $f(g(-2)) = f(-1) = 1 + a$ •² $a = -2$
(b)	**ans: x = -2, 0, 2 (5 marks)**	
	•¹ substitutes •² simplifies •³ equates to 2 •⁴ factorises •⁵ solves for x	•¹ $f(f(x)) = (x^2-2)^2 - 2$ •² $x^4 - 4x^2 + 2$ •³ $x^4 - 4x^2 + 2 = 2; x^4 - 4x^2 = 0$ •⁴ $x^2(x^2-4) = 0$ •⁵ x = -2, 0, 2
5(a)	**ans: x = 1 (2 marks)**	
	•¹ realises y = 0; equates to 0 •² solves for x	•¹ $3x^2 - 6x + 3 = 0$ •² $3(x-1)^2 = 0; x = 1$
(b)	**ans: b = 2 (5 marks)**	
	•¹ integrates expression •² substitutes values •³ simplifies, equates to 1, rearranges •⁴ uses synthetic division to solve •⁵ realises one solution; discards $b^2 - b + 1$	•¹ $[x^3 - 3x^2 + 3x]_1^b$ •² $(b^3 - 3b^2 + 3b) - (1 - 3 + 3)$ •³ $b^3 - 3b^2 + 3b - 2 = 0$ •⁴ $\begin{array}{r\|rrrr} 2 & 1 & -3 & 3 & -2 \\ & & 2 & -2 & 2 \\ \hline & 1 & -1 & 1 & 0 \end{array}$ •⁵ $b = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
6	**ans: 58** **(4 marks)**	
	•[1] knows to take logs	•[1] $\log 2^{-0.04t} = \log 0\cdot 2$
	•[2] releases the power	•[2] $-0\cdot 04t\log 2 = \log 0\cdot 2$
	•[3] makes t the subject	•[3] $t = \dfrac{\log 0\cdot 2}{(-0\cdot 04\log 2)}$ (or equivalent)
	•[4] answer + rounding	•[4] $t = 58$
7(a)	**ans: S(6,3,-12)** **(3 marks)**	
	•[1] for vector algebra	•[1] $\mathbf{p} - \mathbf{q} = 2(\mathbf{s} - \mathbf{r}) \Rightarrow \mathbf{p} - \mathbf{q} + 2\mathbf{r} = 2\mathbf{s}$ (or equiv.)
	•[2] for substituting values	•[2] $\begin{pmatrix}6\\-1\\4\end{pmatrix} - \begin{pmatrix}10\\3\\-2\end{pmatrix} + \begin{pmatrix}16\\10\\-22\end{pmatrix} = 2\,\mathbf{s}$
	•[3] answer	•[3] S(6,3,-12)
(b)	**ans: proof** **(3 marks)**	•[1] statement made <u>or</u> implied ($\vec{PS}.\vec{SR} = 0$)
	•[1] knows scalar product = 0 (stated or implied)	•[2] $\vec{PS} = \begin{pmatrix}0\\4\\-8\end{pmatrix}$, $\vec{SR} = \begin{pmatrix}2\\2\\1\end{pmatrix}$ (or equivalent)
	•[2] finds both displacements	
	•[3] calculation	•[3] $\vec{PS}.\vec{SR} = 0 + 8 + (-8) = 0$ $\therefore$ right angle
8(a)	**ans: Proof** **(1 mark)**	
	•[1] uses trig ratios and equates	•[1] $\cos x = \dfrac{AB}{4}$, $\sin x = \dfrac{BC}{6}$ $4\cos x^\circ + 6\sin x^\circ = 6\cdot 72$
(b)	**ans: 35°** **(5 marks)**	
	•[1] recognising as $k$cos(… equation	•[1] $4\cos x + 6\sin x = k\cos(x - \alpha)$ (or equivalent)
	•[2] finds k	•[2] $k^2 = 4^2 + 6^2$ $\therefore$ $k = \sqrt{52}$
	•[3] finds α	•[3] $\tan\alpha = \dfrac{6}{4}$, $\alpha = 56\cdot 3^\circ$
	•[4] finds first solution	•[4] $\cos(x - 56\cdot 3) = \frac{6\cdot 72}{\sqrt{52}}$ $\therefore$ $x - 56\cdot 3 = 21\cdot 3$ $x = 77\cdot 6^\circ$
	•[5] finds 2nd solution and decides correct ans.	•[5] *or* $x - 56\cdot 3 = 338\cdot 7$ $\therefore$ $x = 395^\circ = 35^\circ$ then decides 35° as < 45

	Give 1 mark for each •	Illustration(s) for awarding each mark
9(a)	**ans:** $\boldsymbol{m}$ **= 2400ml** **(4 marks)**	
	•[1] knows to differentiate and equate to 0	•[1] $H'(m) = 0$
	•[2] differentiates	•[2] $4 - \frac{1}{600}m = 0$
	•[3] solves for x	•[3] $m = 2400$
	•[4] justifies maximum	•[4] table of values; second derivative
(b)	**ans: 4800 feet** **(1 mark)**	
	•[1] knows to sub into function and evaluate	•[1] $4(2400) - \frac{(2400)^2}{1200} = 4800\text{feet}$

Total: 60 marks