[C100/SQP248]

Higher Mathematics

Time: 1 hour 10 minutes

NATIONAL QUALIFICATIONS

Units 1, 2 and 3
Paper 1
(Non-calculator)
Specimen Question Paper **(Revised)**
for use in and after 2004

Read Carefully

1 **Calculators may NOT be used in this paper**.

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH QUALIFICATIONS AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

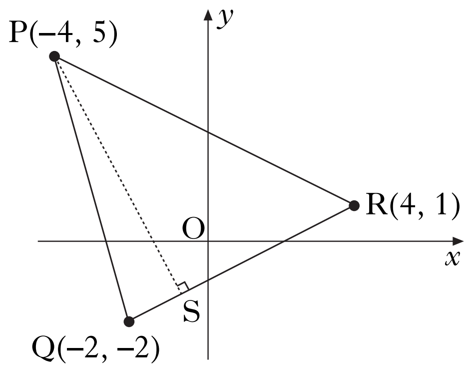
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted. *Marks*

1. P(−4, 5), Q(−2, −2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **(4)**



2. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}3u_n + 5$ with first term u_1.

(*a*) Explain why this sequence has a limit as n tends to infinity. **(1)**

(*b*) Find the **exact** value of this limit. **(2)**

3. (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. **(4)**

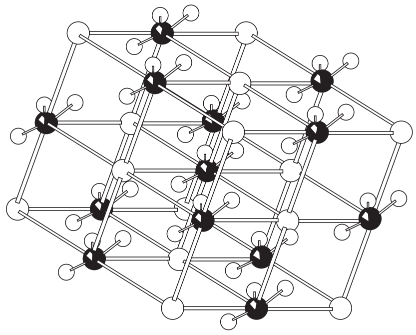
(*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. **(2)**

(*c*) Find the stationary points of $y = f(x)$ and determine the nature of each. **(5)**

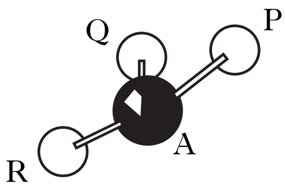
(*d*) Sketch the graph of $y = f(x)$. **(1)**

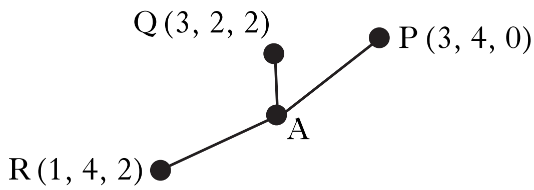
4. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x + 30)°$ is $\dfrac{4\sqrt{3}+3}{10}$. **(4)**

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.





(*a*) Show that the cosine of angle PQR is $\frac{1}{2}$. **(7)**

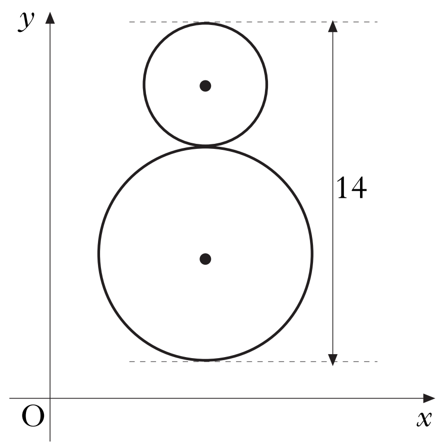
(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2 : 1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(7)**

6. A bakery firm makes ginger-bread men each 14 cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis.

Find the equation of the "head". **(6)**



Marks

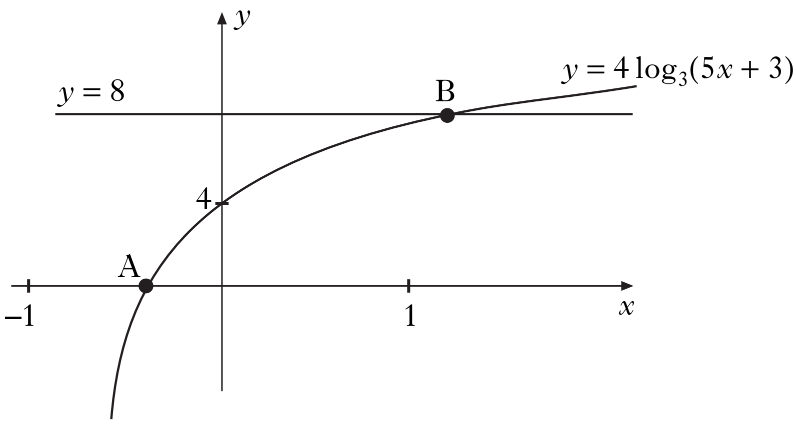
7. Find the value of $\int_1^2 \frac{u^2+2}{2u^2}\,du$. **(7)**

8. Sketch the graph of $y = 2\sin(x - 30)°$ for $0 \leq x < 360$. **(4)**

9. Find $\frac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$. **(3)**

10. Part of the graph of $y = 4\log_3(5x + 3)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.

Find the x-coordinate of B. **(3)**



[*END OF SPECIMEN QUESTION PAPER*]

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[C100/SQP248]

Higher Mathematics
Units 1, 2 and 3
Paper 2
Specimen Question Paper **(Revised)**
for use in and after 2004

Time: 1 hour 30 minutes

NATIONAL QUALIFICATIONS

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SCOTTISH QUALIFICATIONS AUTHORITY
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$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2\sin \mathrm{A} \cos \mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives:

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All questions should be attempted. *Marks*

1. ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC. **(4)**

2. Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees.

A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees.

Which pesticide will be more effective in the long term? **(6)**

3. (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants. **(3)**

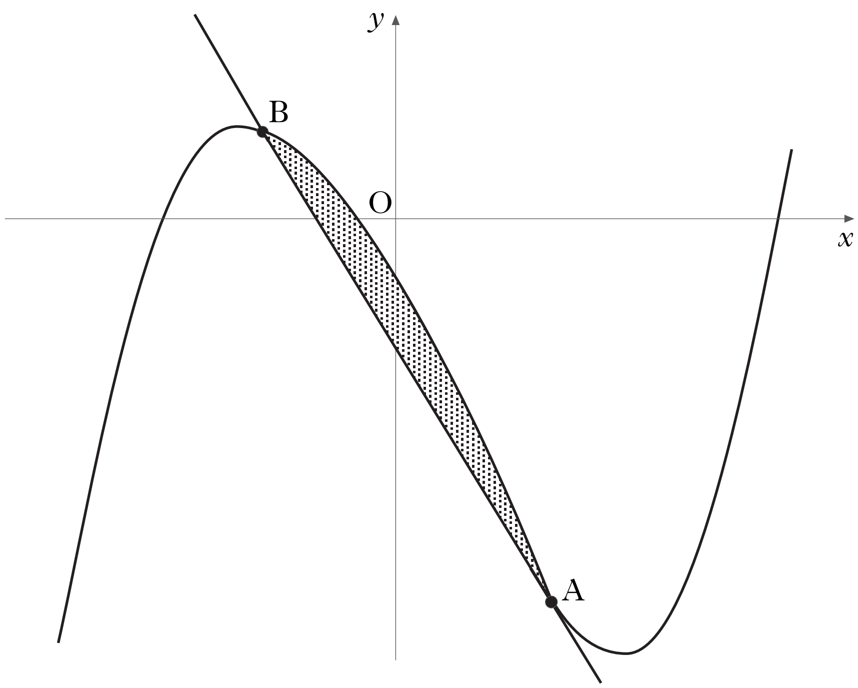
(*b*) Hence, or otherwise, find the coordinates of the turning point of the function f. **(1)**

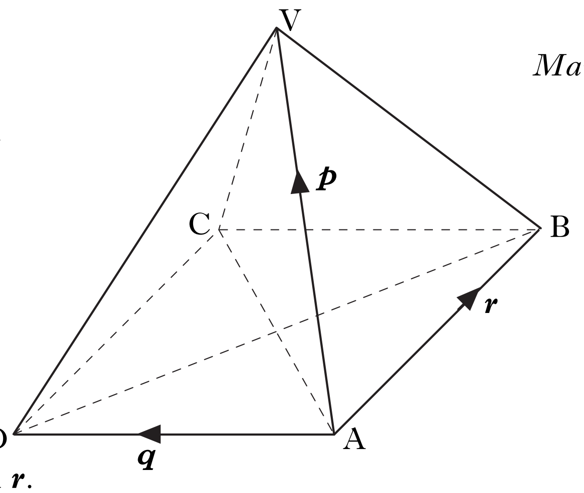
4. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

(*a*) Find the equation of the tangent at A. **(4)**

(*b*) Hence find the coordinates of B. **(5)**

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **(5)**





Marks

5. VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.

$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$.

(*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$.

(ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. **(4)**

(*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$.

(ii) Hence show that angle CVA is 90°. **(4)**

6. $f(x) = 2\cos x° + 3\sin x°$.

(*a*) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$. **(4)**

(*b*) Hence solve $f(x) = 0{\cdot}5$ for $0 \leq x < 360$. **(3)**

(*c*) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the x-axis for $0 \leq x < 360$. **(2)**

7. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **(3)**

(*b*) **Hence**

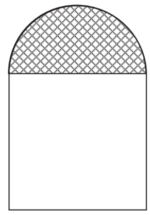
(i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$

(ii) solve this equation in the interval $0 \leq x < 90$. **(4)**

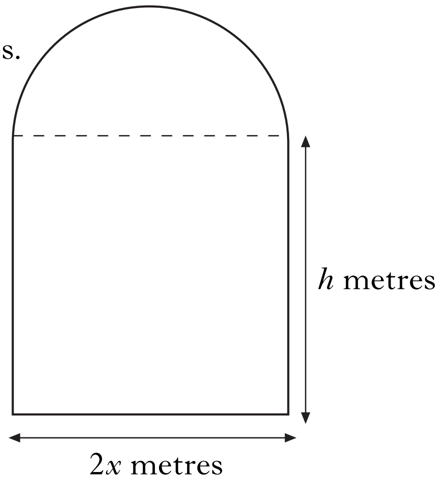
8. The roots of the equation $(x - 1)(x + k) = -4$ are equal.
Find the values of k. **(5)**

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.

The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.



(a) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(b) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(6)**

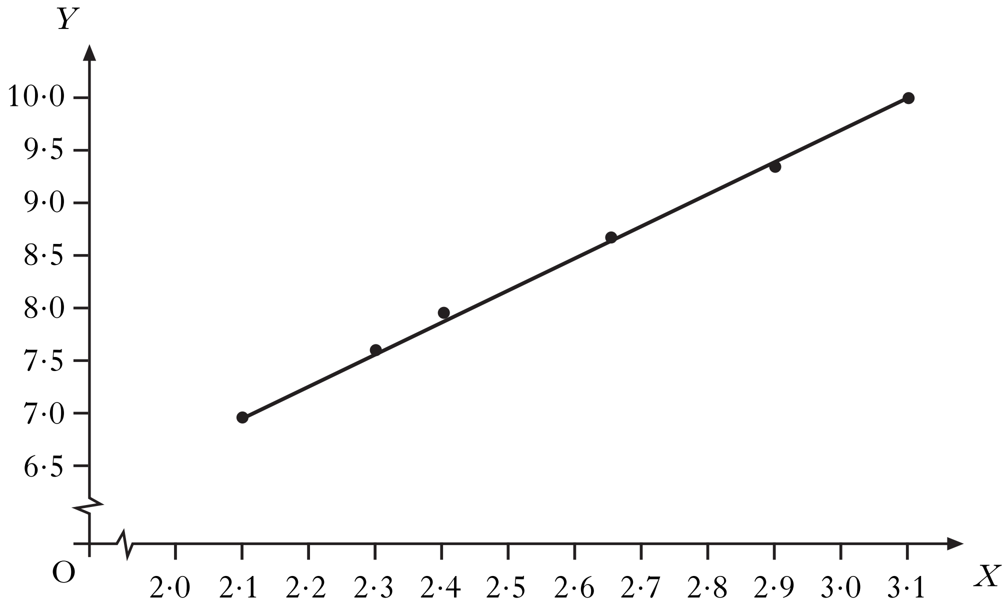
Marks

10. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

$X\ (= \log_e x)$	2·10	2·31	2·40	2·65	2·90	3·10
$Y\ (= \log_e y)$	7·00	7·60	7·92	8·70	9·38	10·00

A graph was drawn and is shown below.



Find the equation of the line in the form $Y = mX + c$. **(3)**

[*END OF SPECIMEN QUESTION PAPER*]

Higher Mathematics

Units 1, 2 and 3

Paper 1

Specimen Marking Instructions **(Revised)**

NATIONAL QUALIFICATIONS

Note: In the Specimen Marking Instructions the Marking Scheme indicates which marks awarded are strategy marks (st), which marks awarded are processing marks (pr) and which marks awarded are interpretation and communication marks (ic).

SCOTTISH QUALIFICATIONS AUTHORITY

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
1	**ans: $y = -2x - 3$ 4 marks** •[1] st: know to get m_{QR} then m_{PS} •[2] pr: use gradient formula •[3] st: know how to find perp. gradient •[4] ic: state equation of st. line	 •[1] strat: find m_{QR} then m_{PS} •[2] $m_{QR} = \frac{1}{2}$ •[3] $m_{PS} = -2$ •[4] PS: $y - 5 = -2(x + 4)$
2	**ans: $-1 < 0{\cdot}3 < 1$, $\frac{50}{7}$ 3 marks** •[1] ic: state condition for limit •[2] st: know how to find limit •[3] pr: complete strategy for exact limit	 •[1] $-1 < 0{\cdot}3 < 1$ •[2] $eg\ L = 0{\cdot}3L + 5$ •[3] $L = \frac{50}{7}$
3a	**ans: $f(1) = 0$, $(x - 4)$, $(x - 1)$ 4 marks** •[1] st: know to find factor of cubic •[2] pr: reach zero at finish •[3] ic: state quadratic factor •[4] pr: complete factorisation	 •[1] $eg \quad \begin{array}{r\|rrrr} 1 & 1 & -6 & 9 & -4 \\ \hline & 1 & & & \end{array}$ •[2] $eg \quad \begin{array}{r\|rrrr} 1 & 1 & -6 & 9 & -4 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & 0 \end{array}$ •[3] $x^2 - 5x + 4$ •[4] $(x - 4)(x - 1)$
3b	**ans: (1,0), (4,0), (0,–4) 2 marks** •[5] ic: x-axis intersections •[6] ic: y-axis intersections	 •[5] (1,0), (4,0) •[6] (0,–4)
3c	**ans: max at (1,0), min at (3,–4) 5marks** •[7] st: know to set derivative = 0 •[8] pr: differentiate •[9] pr: find stationary points •[10] st: know how to test nature •[11] ic: complete nature test	 •[7] $\frac{dy}{dx} = 0$ •[8] $3x^2 - 12x + 9$ •[9] (1,0), (3,–4) •[10] $eg \quad \begin{array}{c\|ccccc} x & \ldots & 1 & \ldots & 3 & \ldots \\ \hline y' & + & 0 & - & 0 & + \end{array}$ •[11] max at (1,0), min at (3,–4)
3d	**ans: sketch 1 mark** •[12] ic: draw sketch	 •[12] sketch

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
4	**ans:** $\mathbf{\dfrac{4\sqrt{3}+3}{10}}$ **4 marks** •1 st: know to expand •2 st: know to use right-angled trig. •3 ic: recall exact values •4 pr: substitute and complete proof	•1 $\sin x^{\circ}\cos 30^{\circ} + \cos x^{\circ}\sin 30^{\circ}$ •2 $\sin x^{\circ} = \frac{4}{5}$ ***and*** $\cos x^{\circ} = \frac{3}{5}$ •3 $\sin 30^{\circ} = \frac{1}{2}$ ***and*** $\cos 30^{\circ} = \frac{\sqrt{3}}{2}$ •4 $\dfrac{4\sqrt{3}+3}{10}$
5a	**ans: proof** **7 marks** •1 ic: find vector components •2 ic: find vector components •3 st: eg know to use scalar product •4 ic: find scalar product •5 pr: find magnitude •6 pr: find magnitude •7 pr: complete proof	•1 $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$ •2 $\overrightarrow{RQ} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$ •3 $\cos PQR = \dfrac{\overrightarrow{PQ}.\overrightarrow{RQ}}{\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{RQ}\rvert}$ •4 $\overrightarrow{PQ}.\overrightarrow{RQ} = 4$ •5 $PQ = \sqrt{8}$ •6 $RQ = \sqrt{8}$ •7 substitution leading to $\frac{1}{2}$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
5b	**ans: M(2,3,2) T$\left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$; proof** **7 marks** •8 ic: find coordinates of M •9 ic: interpret ratio •10 pr: find appropriate vector •11 pr: find appropriate vector •12 pr: complete calc. of coordinates of T •13 st: know how to find distance in 3D •14 pr: complete proof	•8 $M = (2,3,2)$ •9 $eg\ \overrightarrow{PT} = \frac{2}{3}\overrightarrow{PM}$ •10 $\overrightarrow{PM} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ •11 $\overrightarrow{PT} = \begin{pmatrix} -\frac{2}{3} \\ -\frac{2}{3} \\ \frac{4}{3} \end{pmatrix}$ •12 $T = \left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$ •13 $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ •14 $PT = 2\sqrt{\frac{2}{3}},\quad QT = 2\sqrt{\frac{2}{3}},\quad RT = 2\sqrt{\frac{2}{3}}$
6	**ans: $(x-5)^2 + (y-13)^3 = 9$ 6 marks** •1 ic: state radius from equ. of circle •2 ic: interpret diagram •3 ic: interpret distance between centres •4 ic: state centre from equ. of circle •5 ic: interpret diagram •6 ic: state equation of circle	•1 $radius_{body} = 4$ •2 $radius_{head} = 3$ •3 distance between centres = 7 •4 $centre_{body} = (5,6)$ •5 $centre_{head} = (5,13)$ •6 $(x-5)^2 + (y-13)^3 = 9$
7	**ans: 1 7 marks** •1 st: know to split into sums/differences •2 pr: write in integrable form •3 pr: write in integrable form •4 pr: integrate •5 pr: integrate •6 pr: substitute limits of integration •7 pr: evaluate	•1 $\frac{u^2}{2u^2} + \frac{2}{2u^2}$ •2 $\frac{1}{2}$ •3 u^{-2} •4 $\frac{1}{2}u$ •5 $-u^{-1}$ •6 $\left(\frac{1}{2}\times 2 - 2^{-1}\right) - \left(\frac{1}{2}\times 1 - 1^{-1}\right)$ •7 1

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
8	**ans: sketch** **4 marks** for $y = a\sin(x + b)$ •1 ic: know "b" represents translation •2 ic: know "a" represents vert. scaling •3 st: know the order of transformations •4 ic: complete sketch	•1 "–30 °" means move $y = \sin x^{\circ}$: $\begin{pmatrix} +30^{\circ} \\ 0 \end{pmatrix}$ •2 "2" means stretch two-fold parallel to y-axis •3 order is "–30 °" then "2" •4 sketch evidence for •1, •2 and •3 may be either stated, or implied by a correct sketch at the •4 stage.
9	**ans:** $-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$ **3 marks** •1 st: know how to deal with √ •2 pr: start differentiation •3 pr: apply chain rule	•1 $(1 + \cos x)^{\frac{1}{2}}$ •2 $\frac{1}{2}(1+\cos x)^{-\frac{1}{2}}$ •3 $\times - \sin x$
10	**ans:** $\frac{6}{5}$ **3 marks** •1 ic: interpret diagram •2 st: know how to solve log equation •3 pr: complete the solving	•1 $\log_3(5x + 3) = 2$ •2 $5x + 3 = 3^2$ •3 $x = \frac{6}{5}$

[*END OF SPECIMEN MARKING INSTRUCTIONS*]

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[C100/SQP248]

Higher
Mathematics
Units 1, 2 and 3
Paper 2
Specimen Marking Instructions **(Revised)**

NATIONAL
QUALIFICATIONS

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QUALIFICATIONS
AUTHORITY

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1	**ans: $y = -2x - 5$** **4 marks** •[1] st: know to use gradient formula •[2] pr: use gradient formula •[3] ic: know parallel lines have equal gradients •[4] ic: state equation of st. line	 •[1] $m = \frac{y_2 - y_1}{x_2 - x_1}$ •[2] $m_{AB} = 2$ •[3] $m_{DC} = 2$ •[4] $y - 11 = 2(x - 8)$
2	**ans: Pestkill** **6 marks** •[1] ic: state the scaling factor •[2] ic: interpret the constant in recurrence relation •[3] ic: state recurrence relation •[4] st: know how to find limit •[5] pr: complete strategy for limits •[6] ic: limit condition & conclusion	 •[1] 0·4 •[2] $au_n + 300$ •[3] $0{\cdot}2u_n + 360$ •[4] *eg* $L = aL + b$ •[5] 500 *and* 450 •[6] limits are valid since $\lvert a \rvert < 1$ in both cases and Pestkill more effective
3a	**ans: $2(x + 2)^2 - 11$** **3 marks** •[1] pr: start with a eg $2(x^2 + 4x)$ •[2] pr: continue for b eg $2(x + 2)^2$ •[3] pr: complete by finding c	 •[1] $a = 2$ •[2] $b = 2$ •[3] $c = -11$
3b	**ans: (–2, 11)** **1 mark** •[4] ic: state turning point of $a(x + b)^2 + c$	 •[4] (–2, 11)

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
4a	**ans: $y = -5x - 3$ 4 marks** •[1] st: know to differentiate •[2] pr: differentiate •[3] pr: evaluate gradient •[4] ic: state equation of tangent	 •[1] $\frac{dy}{dx} =$ •[2] $\frac{dy}{dx} = 3x^2 - 2x - 6$ •[3] $m_{tgt} = -5$ •[4] $y + 8 = -5(x - 1)$
4b	**ans: B = (–1,2) 5 marks** •[5] st: know how to find intersection •[6] pr: produce cubic in standard form •[7] st: know how to solve cubic •[8] pr: achieve linear and quadratic factor •[9] ic: interpret coordinates **ans: area $= 1\frac{1}{3}$ 5 marks** •[10] st: know to subtract: "upper–lower" •[11] ic: state limits of integration •[12] pr: integrate •[13] ic: interpret limits •[14] pr: evaluate limits	 •[5] attempt to simplify and equate y's •[6] $x^3 - x^2 - x + 1 = 0$ •[7] evidence of eg synthetic division •[8] $(x-1)(x^2 - 1)$ •[9] B = (–1,2) •[10] $\int (x^3 - x^2 - 6x - 2) - (-5x - 3)dx$ •[11] $\int_{-1}^{1}$ •[12] $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$ •[13] $\left(\frac{1}{4}.1^4 - \frac{1}{3}.1^3 - \frac{1}{2}.1^2 + 1\right)$ $-\left(\frac{1}{4}(-1)^4 - \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 + (-1)\right)$ •[14] $1\frac{1}{3}$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
5a	**ans: 9** **4 marks** •1 ic: interpret lengths and angle •2 pr: evaluate scalar product •3 st: know to use distributive law •4 pr: evaluate scalar product & complete	 •1 $\lvert \boldsymbol{p} \rvert = \lvert \boldsymbol{q} \rvert = 3$, $\quad V\hat{A}D = 60°$ •2 $\frac{9}{2}$ •3 $\boldsymbol{p.q} + \boldsymbol{p.r}$ •4 9
5b	**ans: proof** **4 marks** •5 ic: interpret 3D representation •6 st: know to use approp. scalar product •7 st: know to use distributive law •8 pr: evaluate scalar product & complete	 •5 $\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r}$ •6 $\boldsymbol{p}.\ (\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r})$ •7 $\boldsymbol{p.p} - \boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ •8 0
6a	**ans: $\sqrt{13}\cos(x - 56\cdot3)°$** **4 marks** •1 ic: state expansion •2 ic: compare & equate coefficients •3 pr: solve for k •4 pr: solve for α	 •1 $k\cos x\cos\alpha + k\sin x\sin\alpha$ explicitly stated •2 $k\cos\alpha = 2$ ***and*** $k\sin\alpha = 3$ explicitly stated •3 $k = \sqrt{13}$ •4 $\alpha = 56\cdot3$
6b	**ans: 138·8, 334·3** **3 marks** •5 st: know how to solve trig. equation •6 pr: solve for $x - \alpha$ •7 pr: complete solving process	 •5 $\cos(x - 56\cdot3)° = \frac{0\cdot5}{\sqrt{13}}$ •6 $x - 56\cdot3 = 82\cdot0,\ 278\cdot0$ •7 $x = 138\cdot3,\ 334\cdot3$
6c	**ans: 146·3°** **2 marks** •8 st: know how to set function = 0 •9 pr: complete solving process	 •8 $\sqrt{13}\cos(x - 56\cdot3)° = 0$ •9 $x = 146\cdot3$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
7a	**ans: proof** **3 marks**	
	•[1] st: know to use approp. trig. rule •[2] st: know to use approp. trig. rule •[3] ic: complete the proof	•[1] substitute $1 - 2\sin^2 x°$ for $\cos 2x°$ •[2] substitute $1 - \sin^2 x°$ for $\cos^2 x°$ •[3] complete proof
7b	**ans: 19·5** **4 marks**	
	•[4] st: know to express in standard form •[5] pr: factorise quadratic •[6] pr: continue solving process •[7] ic: complete solving process	•[4] $3\sin^2 x° + 2\sin x° - 1 = 0$ •[5] $(3\sin x° - 1)(\sin x° + 1) = 0$ •[6] $x = \frac{1}{3}$ and $x = -1$ •[7] $x = 19 \cdot 5$ and no other answers
8	**ans: $k = -5$ or 3** **5 marks**	
	•[1] st: know to express in standard form •[2] st: know condition for equal roots •[3] pr: apply the strategy •[4] pr: start the solving process •[5] pr: complete the solving process	•[1] $x^2 + kx - x + 4 - k = 0$ •[2] $b^2 - 4ac = 0$ •[3] $(k-1)^2 - 4(4-k)$ •[4] $k^2 + 2k - 15 = 0$ •[5] $k = -5$, $k = 3$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
9ai	**ans: $h = \frac{1}{2}(10 - \pi x - 2x)$ 2 marks** •¹ st: know to form equ. for perimeter •² pr: make h the subject	 •¹ *eg* $2h + 2x +$ semicircle $= 10$ •² $h = \frac{1}{2}(10 - \pi x - 2x)$
9aii	**ans: proof 2 marks** •³ st: know how to set up equ. for L •⁴ ic: complete proof	 •³ $L = 2 \times 2xh + \frac{1}{2}\pi x^2$ •⁴ $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$ $L = 20x - 2\pi x^2 - 4x \ + \frac{1}{2}\pi x^2$
9b	**ans: $x = \frac{20}{3\pi + 8}$, $h = \frac{5(\pi + 4)}{3\pi + 8}$ 6 marks** •⁵ st: know to differentiate •⁶ pr: differentiate •⁷ st: know that max. means $L' = 0$ •⁸ pr: solve $L' = 0$ •⁹ st: know to check nature of max/min •¹⁰ ic: complete evaluation	 •⁵ $\frac{dL}{dx} =$ •⁶ $\frac{dL}{dx} = 20 - 8x - 3\pi x$ •⁷ $\frac{dL}{dx} = 0$ •⁸ $x = \frac{20}{3\pi + 8} = x_0 (= 1 \cdot 148)$ •⁹ *eg* x x_0^- x_0 x_0^+ L' $+$ 0 $-$ maximum at x_0 •¹⁰ $h = \frac{5\pi + 20}{3\pi + 8} (= 2 \cdot 049)$
10	**ans: Y = 3X + 0·7 3 marks** •¹ ic: interpret gradient from graphs •² st: know how to find "c" •³ pr: complete evaluation	 •¹ $m = 3$ •² *eg* $7{\cdot}00 = 3 \times 2{\cdot}10 + c$ •³ *eg* $Y = 3X + 0{\cdot}7$

[*END OF SPECIMEN MARKING INSTRUCTIONS*]