Homework 1

1) The position of a remote controlled model boat on a pond, relative to a rectangular coordinate system with origin O, is given by

$$\mathbf{r} = (3t^2 - 12t + 5)\mathbf{i} + (4t - t^2)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the Ox and Oy directions respectively, t is time measured in seconds and distances are measured in metres.

Calculate the distance of the boat from the origin O when it comes to instantaneous rest. **4**

2) A particle has velocity $3t(2 - t)\mathbf{j}$ where $\mathbf{j}$ is the unit vector in the direction of motion. The time t is measured in seconds from the start of the motion and the displacement is measured in metres. Initially the particle is at the point with position vector $3\mathbf{j}$ relative to the origin O. Calculate the distance of the particle from O when the velocity is a maximum. **4**

3) The position of a model boat P, relative to a rectangular coordinate system with origin O, is given by

$$\mathbf{r}_P = t^2\mathbf{i} + 4t\mathbf{j}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions respectively, t is the time measured in seconds and distances are measured in metres.

The acceleration of a second boat Q is given by

$$\mathbf{a}_Q = 2\mathbf{i} + (4\pi \sin 2\pi t)\mathbf{j}.$$

Given that boat Q is initially at rest, find the first two times when the boats have the same velocity. **5**

4) Relative to a rectangular coordinate system, the position of an ice skater at time t seconds is

$$\mathbf{r}(t) = \left(\frac{1}{3}t^3 - 4t^2\right)\mathbf{i} - (2t^2 - 1)\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are the unit vectors in the x, y directions respectively and distances are measured in metres.

Find the speed of the ice skater at the instant when the acceleration is parallel to the y-axis. **5**

5) The velocity of an ice skater relative to a rectangular coordinate system with origin O, is given by

$$\mathbf{v} = 3(t^2 - 4t + 2)\mathbf{i} + 4\mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the Ox and Oy directions, t seconds is the time and the speed is measured in $\mathrm{m\,s^{-1}}$. Initially the skater has position vector $-4\mathbf{j}$.

(*a*) Find the time at which the acceleration is instantaneously equal to zero. **2**

(*b*) Calculate the distance of the skater from O when the acceleration is instantaneously equal to zero. **4**