Homework 1

1) $s = (3t^2 - 12t + 5)\underline{i} + (4t - t^2)\underline{j}$

$v = (6t - 12)\underline{i} + (4 - 2t)\underline{j}$ ✓

$v = 0$ when $6t - 12 = 0$ and $4 - 2t = 0$

$t = 2$ $t = 2$ ✓

⇒ stationary when $t = 2$

$s = (3t^2 - 12t + 5)\underline{i} + (4t - t^2)\underline{j}$

$t = 2 \Rightarrow s = -7\underline{i} + 4\underline{j}$ ✓

so distance $|s| = \sqrt{7^2 + 4^2} = \underline{\sqrt{65}\text{ metres}}\ (= 8.06\text{m})$ ✓

2) $v = 3t(2 - t)\underline{j}$

$v = (6t - 3t^2)\underline{j}$

max velocity when acceleration $= 0$

$a = (6 - 6t)\underline{j}$ ✓ $a = 0$ when $6 - 6t = 0$

$t = 1$ ✓

$s = (3t^2 - t^3)\underline{j} + C$

at $t = 0$ $s = 3\underline{j}$ $3\underline{j} = 0 + C$

$C = 3\underline{j}$

$s = (3t^2 - t^3 + 3)\underline{j}$ ✓

$t = 1 \Rightarrow s = 5\underline{j}$ so distance $= \underline{5\text{ metres}}$ ✓

3) $r_P = t^2\underline{i} + 4t\underline{j}$

$v_P = 2t\underline{i} + 4\underline{j}$ ✓

$a_Q = 2\underline{i} + (4\pi \sin 2\pi t)\underline{j}$

$v_Q = 2t\underline{i} - \frac{4\pi \cos 2\pi t}{2\pi}\underline{j} + C$

$v_Q = 2t\underline{i} - 2\cos 2\pi t\,\underline{j} + C$

at $t=0$ $v=0$ $\Rightarrow$ $0 = -2\underline{j} + C$

$C = 2\underline{j}$

$v_Q = 2t\underline{i} + (2 - 2\cos 2\pi t)\underline{j}$ ✓

so $v_P = v_Q$ when $\underline{i}$ and $\underline{j}$ components are equal.

$\underline{j}$ components $\Rightarrow$ $4 = 2 - 2\cos 2\pi t$ ✓

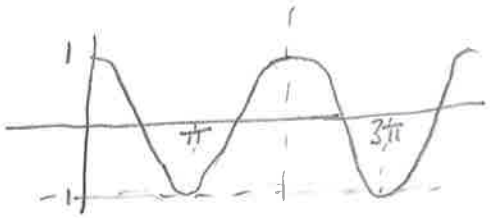
$2\cos 2\pi t = -2$

$\cos 2\pi t = -1$

$2\pi t = \pi, 3\pi$

$t = \frac{\pi}{2\pi}, \frac{3\pi}{2\pi}$

$t = \frac{1}{2}, \frac{3}{2}$ ✓



4) $s(t) = \left(\frac{1}{3}t^3 - 4t^2\right)\underline{i} - (2t^2 - 1)\underline{j}$

$v(t) = (t^2 - 8t)\underline{i} - 4t\underline{j}$ ✓

$a(t) = (2t - 8)\underline{i} - 4\underline{j}$ ✓

acceleration is parallel to the y-axis when $\underline{i}$ component $= 0$

$\Rightarrow$ $2t - 8 = 0$

$t = 4$ ✓

$v = (t^2 - 8t)\underline{i} - 4t\underline{j}$

$t = 4$ $\Rightarrow$ $v = -16\underline{i} - 16\underline{j}$ ✓ so speed $= \sqrt{16^2 + 16^2} = 16\sqrt{2}\ ms^{-1}$ ✓

$= 22.6\ ms^{-1}$

5) a) $\underline{v} = 3(t^2 - 4t + 2)\underline{i} + 4\underline{j}$

$\underline{v} = (3t^2 - 12t + 6)\underline{i} + 4\underline{j}$

$\underline{a} = (6t - 12)\underline{i}$ ✓

$a = 0$ when $6t - 12 = 0$

$\underline{t = 2}$ ✓

b) $\underline{v} = (3t^2 - 12t + 6)\underline{i} + 4\underline{j}$ ✓

$\underline{s} = (t^3 - 6t^2 + 6t)\underline{i} + 4t\underline{j} + \underline{C}$

at $t = 0$ $\underline{s} = -4\underline{j} \Rightarrow -4\underline{j} = \underline{C}$

$\underline{s} = (t^3 - 6t^2 + 6t)\underline{i} + (4t - 4)\underline{j}$ ✓

at $t = 2$ $\underline{s} = -4\underline{i} + 4\underline{j}$

$\Rightarrow$ distance $= \sqrt{4^2 + 4^2} = \underline{4\sqrt{2}\,m} (= 5.66\,m)$ ✓