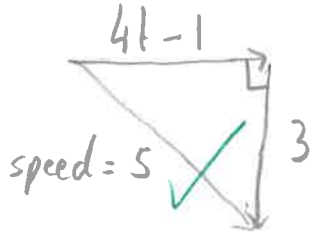
Homework 3 solutions

1) $r(t) = (2t^2 - t)\underline{i} - (3t+1)\underline{j}$

$v(t) = (4t-1)\underline{i} - 3\underline{j}$ ✓



so using pythagoras $4t - 1 = 4$ ✓

$4t = 5$

$t = 1.25s$ ✓

[or $(4t-1)^2 + 3^2 = 5^2$

$(4t-1)^2 = 16$

$4t - 1 = \pm 4$

$4t = 5$ since $t > 0$]

$t = 1.25s$

2) car

$a = a$

$v = at + c$

at $t=0$ $v=0$ $\Rightarrow$ $c = 0$

$v = at$

$s = \frac{1}{2}at^2 + c$

at $t=0$ $s=0$ $\Rightarrow$ $c=0$

$\underline{s_c = \frac{1}{2}at^2}$ ✓

lorry

$a = 0$

$v = u$

$s = ut + c$

at $t=0$ $s=0$ $\Rightarrow$ $c=0$

$\underline{s_L = ut}$ ✓

car and lorry will draw level when $S_C = S_L$

$$\frac{1}{2}at^2 = Ut$$

$$\frac{1}{2}at^2 - Ut = 0$$

$$t\left(\frac{1}{2}at - U\right) = 0$$

$t = 0$ and $\frac{1}{2}at - U = 0$

$$t = \frac{2U}{a} \checkmark$$

at $t = \frac{2U}{a}$

$$S_L = Ut$$

$$S_L = U\left(\frac{2U}{a}\right)$$

$$S_L = \frac{2U^2}{a} \checkmark = S_C$$

3) $u = 2$

$S = 120$

$a = 0.25$

$t = T$

$$S = ut + \frac{1}{2}at^2$$

$$120 = 2T + \tfrac{1}{2} \times 0.25T^2 \checkmark$$

$$120 = 2T + \frac{1}{8}T^2$$

$$960 = 16T + T^2 \checkmark$$

$$T^2 + 16T - 960 = 0$$

$$T^2 + 16T - 960 = 0$$

$$(T + 40)(T - 24) = 0 \checkmark$$

$T = -\cancel{40}$, 24 ✓

$V = u + at$

$V = 2 + \frac{1}{4} \times 24$

$V = 8\,ms^{-1}$ ✓

4) **Lift**

$a = \frac{1}{8}g$

$v = \frac{1}{8}gt + C$

at $t=0$ $v=0$ $\Rightarrow$ $C=0$

$v = \frac{1}{8}gt$

$s = \frac{1}{16}gt^2 + C$

at $t=0$ $s=0$ $\Rightarrow$ $C=0$

$\underline{s_L = \frac{1}{16}gt^2 j}$ ✓

light bulb

$a = -g$

$v = -gt + C$

at $t=0$ $v=0$ $\Rightarrow$ $C=0$

$v = -gt$

$s = -\frac{1}{2}gt^2 + C$

at $t=0$ $s=2$ $\Rightarrow$ $C=2$

$\underline{s_b = (2 - \frac{1}{2}gt^2) j}$ ✓

bulb relative to lift $= s_b - s_L$ ✓

$= (2 - \frac{1}{2}gt^2) j - \frac{1}{16}gt^2 j$

$= \underline{(2 - \frac{9}{16}gt^2) j}$

b) when bulb hits the floor the relative distance between them $= 0$

$\Rightarrow \quad 2 - \frac{9}{16}gt^2 = 0$ ✓

$\frac{9}{16}gt^2 = 2$

$\underline{t = 0.602s}$ ✓

at $t = 0.602s$ $\quad s_b = 2 - \frac{1}{2}gt^2$

$= \underline{0.222m}$ $\Rightarrow$ bulb has fallen $2 - 0.222 = \underline{1.78m}$ ✓

5) passenger aircraft

position at 0900 $S = -100\underline{i} + 250\underline{j}$

position at 0930 $S = -100\underline{i} + 150\underline{i} + 250\underline{j} + 200\underline{j}$

$S = 50\underline{i} + 450\underline{j}$ ✓

$V_p = 300\underline{i} + 400\underline{j}$

$S_p = 300t\underline{i} + 400t\underline{j} + C$

at 0930 $(t=0)$ $S_p = 50\underline{i} + 450\underline{j} \Rightarrow C = 50\underline{i} + 450\underline{j}$

$S_p = (300t + 50)\underline{i} + (400t + 450)\underline{j}$ ✓

military aircraft

position at 0930 $(t=0)$ $S = -100\underline{i} + 400\underline{j}$

$V_M = 600\underline{i} + 500\underline{j}$

$S_M = 600t\underline{i} + 500t\underline{j} + C$ at $t=0$ $S = -100\underline{i} + 400\underline{j} = C$ ✓

$S_M = (600t - 100)\underline{i} + (500t + 400)\underline{j}$

collision if $\underline{i}$ and $\underline{j}$ components are equal at same time

$\underline{i}$ components

$300t + 50 = 600t - 100$ ✓

$300t = 150$

$t = 0.5$

$\underline{j}$ components

$400t + 450 = 500t + 400$

$100t = 50$

$t = 0.5$

hence collision will occur at $t = 0.5$ or at 1000hours. ✓