Homework 4 solutions

1)

horizontal distance

$s = (u\cos\theta)t$

$70 = (30\cos\theta) \times 3$

$70 = 90\cos\theta$

$\cos\theta = 70/90$

$\theta = 38.9^\circ$

vertical distance

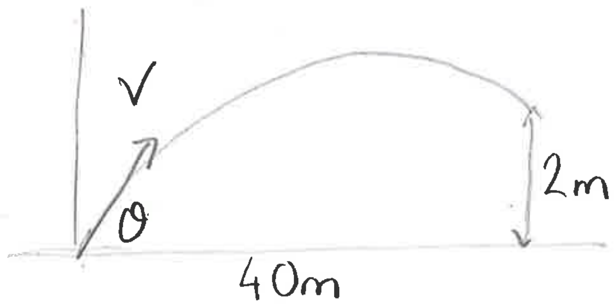
$s = ut + \frac{1}{2}at^2$

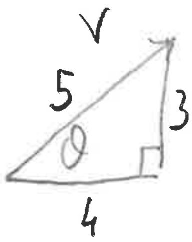
so height $H = (30\sin\theta) \times 3 - \frac{1}{2}g \times 3^2$

so height $H = (30\sin 38.9) \times 3 - \frac{1}{2}g \times 3^2$

$= 12.5\text{m}$

2)





$\tan\theta = \frac{3}{4}$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

horizontal distance

$s = (u\cos\theta) \times t$

$40 = u \times \frac{4}{5} \times t$

$50 = ut$

vertical distance

$s = ut + \frac{1}{2}at^2$

$2 = (u\sin\theta)t - \frac{1}{2}gt^2$

$2 = u \times \frac{3}{5} \times t - \frac{1}{2}gt^2$

$2 = \frac{3}{5} \times ut - \frac{1}{2}gt^2$

$2 = \frac{3}{5} \times 50 - \frac{1}{2}gt^2$

$2 = 30 - \frac{1}{2}gt^2$

$\frac{1}{2}gt^2 = 28 \Rightarrow t = 2.39\text{s}$

$ut = 50$

$u = \frac{50}{2.39}$

$u = 20.9\text{ms}^{-1}$

3) <u>horizontal distance</u>

$$s = (u\cos\theta)t$$

<u>vertical distance</u>

$$s = (u\sin\theta)t - \tfrac{1}{2}gt^2$$

at max height $v = 0$

$u = u\sin\theta$

$a = -g$

$t = ?$

$$v = u + at$$
$$0 = u\sin\theta - gt$$
$$t = \frac{u\sin\theta}{g}$$

so max height

$$h = (u\sin\theta)t - \tfrac{1}{2}gt^2$$
$$h = u\sin\theta\left(\frac{u\sin\theta}{g}\right) - \tfrac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$$
$$h = \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$$
$$h = \frac{u^2\sin^2\theta}{2g}$$

at range total time $T = 2t = \dfrac{2u\sin\theta}{g}$

$$R = u\cos\theta\left(\frac{2u\sin\theta}{g}\right)$$
$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

(3)

$R = 10 \times h$

so $$\frac{2u^2 \sin\theta \cos\theta}{g} = 10 \times \frac{u^2 \sin^2\theta}{2g}$$ ✓

$$\frac{2\cancel{u^2} \sin\theta \cos\theta}{\cancel{g}} = \frac{10\cancel{u^2} \sin^2\theta}{2\cancel{g}}$$

$$\cancel{2}^{1}\cancel{\sin\theta} \cos\theta = \cancel{10}^{5} \sin^{\cancel{2}\,1}\theta$$ ✓

$$\cos\theta = 5\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{2}{5}$$ ✓

$$\tan\theta = 2/5$$ ✓

$$\underline{\theta = 21.8°}$$ ✓

4a) <u>horizontal distance</u>

$$s = ut$$

$$s = (v\cos\alpha)t$$ ✓

so total time $T = \dfrac{2v\sin\alpha}{g}$ ✓

so Range $R = v\cos\alpha\left(\dfrac{2v\sin\alpha}{g}\right)$ ✓

$$R = \frac{v^2}{g} \times 2\sin\alpha\cos\alpha$$

$$\underline{R = \frac{v^2}{g} \times \sin 2\alpha}$$

max height when $v = 0$

$u = v\sin\alpha$

$a = -g$

$t = ?$

$$v = u + at$$

$$0 = v\sin\alpha - gt$$

$$t = \frac{v\sin\alpha}{g}.$$ ✓

b) so $L < R < 2L$

$$L < \frac{v^2}{g} \times \sin 2\alpha < 2L \qquad \alpha = 15^\circ$$

$$L < \frac{v^2}{g} \times \sin 30 < 2L$$

$$L < \frac{v^2}{2g} < 2L \qquad (\div L)$$

$$1 < \frac{v^2}{2gL} < 2 \qquad (\times 2)$$

$$2 < \frac{v^2}{gL} < 4$$

$$\Rightarrow \quad \sqrt{2} < \sqrt{\frac{v^2}{gL}} < 2$$

$$\Rightarrow \quad \underline{\sqrt{2} < \frac{v}{\sqrt{gL}} < 2}$$

5) (5)

horizontal distance

$$s = (u\cos\theta)t$$

$$3H = (V\cos 30)t$$

$$t = \frac{3H}{V\cos 30}$$

vertical distance

$$s = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$H = (V\sin 30)t - \frac{1}{2}gt^2$$

$$H = V\sin 30\left(\frac{3H}{V\cos 30}\right) - \frac{g}{2}\left(\frac{3H}{V\cos 30}\right)^2$$

$$H = 3H\tan 30 - \frac{g}{2}\times\frac{12H^2}{V^2}$$

$$H = \sqrt{3}H - \frac{6gH^2}{V^2}$$

$$\frac{6gH^2}{V^2} = \sqrt{3}H - H$$

$$\frac{6gH}{V^2} = \sqrt{3} - 1$$

$$V^2 = \frac{6gH}{\sqrt{3}-1}$$

$$V = \sqrt{\frac{6gH}{\sqrt{3}-1}}$$ to hit the top of the wall

so to get over the wall

$$V > \sqrt{\frac{6gH}{\sqrt{3}-1}}$$