Homework 5

1)

A particle, initially at rest, is projected from the origin with acceleration $(12 - 3t^2)\,\mathbf{i}\ \mathrm{m\,s^{-2}}$, where $\mathbf{i}$ is the unit vector in the direction of motion, and t is the time measured in seconds from the start of the motion.

Determine the position of the particle when it next comes to rest. **4**

2)

Motorcyclist A has uniform acceleration $-2\mathbf{j}\ \mathrm{m\,s^{-2}}$, initial velocity $\mathbf{i}\ \mathrm{m\,s^{-1}}$ and initial position $-\mathbf{i}$ metres relative to a rectangular coordinate system with unit vectors $\mathbf{i}$, $\mathbf{j}$ in the x, y directions respectively.

(a) Find the position $\mathbf{r}_A(t)$ of the motorcyclist A at time t seconds, where t is measured from the start of the motion. **2**

The position of a second motorcyclist B relative to the same coordinate system as A is

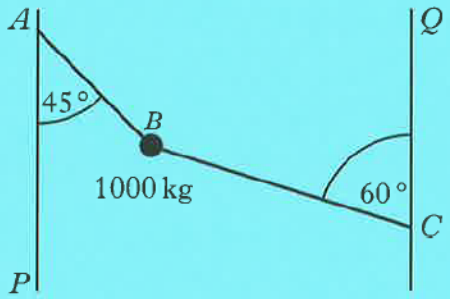
$$\mathbf{r}_B(t) = (2t - 3)\mathbf{i} + (1 - t^2)\mathbf{j}.$$

(b) (i) Find the position of A relative to B. **1**

(ii) Calculate the minimum distance between the motorcyclists. **3**

3)

On a construction site, a 1000 kg concrete block is supported in equilibrium by two light inextensible chains AB and BC, attached to the block at B, as shown below.



PA and CQ are vertical with $\angle PAB = 45°$ and $\angle BCQ = 60°$. The tensions in the chains over sections AB and BC are denoted by T_1 and T_2 respectively.

(a) By resolving the forces horizontally, find a relationship between T_1 and T_2. **2**

(b) Calculate the tension T_2. **3**

4) A missile is launched from ground level with speed $V\,\mathrm{m\,s^{-1}}$ at an angle of $30°$ to the horizontal.

(*a*) Show that the height y metres of the missile at time t is given by

$$y = \tfrac{1}{2}t\,(V - gt),$$

where $g\,\mathrm{m\,s^{-2}}$ is the magnitude of the acceleration due to gravity, and t is measured in seconds from the moment of launch. **3**

(*b*) Find the maximum height H attained by the missile, giving your answer in terms of V and g. **2**

(*c*) A missile is detected on radar if $y \geq \tfrac{1}{4}H$. Show that the missile appears on radar for $\dfrac{\sqrt{3}V}{2g}$ seconds. **5**

5) Differentiate with respect to x:

$$y = \frac{\cos x}{1 - \sin x}, \qquad x \neq (2n + \tfrac{1}{2})\pi.$$

Simplify your answer as far as possible. **3**