National
Qualifications
SPECIMEN ONLY

SQ25/AH/01

Mathematics of Mechanics

Date — Not applicable

Duration — 3 hours

Total marks — 100

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate. Any rounded answer should be accurate to three significant figures (or one decimal place for angles in degrees) unless otherwise stated.

Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a\sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax) + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$

Total marks — 100 **MARKS**

Attempt ALL questions

Candidates should observe that $g\,\text{m s}^{-2}$ denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be $9{\cdot}8\,\text{m s}^{-2}$.

1. A curling stone, P, of mass 18 kg is moving with velocity $\begin{pmatrix} 0 \\ -1{\cdot}1 \end{pmatrix}\,\text{m s}^{-1}$ relative to a suitable set of coordinate axes. It collides with a stationary curling stone, Q, of mass 20 kg. Q then moves off with velocity $\begin{pmatrix} 0{\cdot}36 \\ -0{\cdot}72 \end{pmatrix}\,\text{m s}^{-1}$.

Calculate the speed with which P travels immediately after impact. **3**

2. Given $y = e^{x^2}\cos x$ find $\dfrac{dy}{dx}$. **3**

3. A cyclist climbs a hill of length 2·4 km and constant gradient 1:25 in 12 minutes. When cycling at a constant speed of $v\,\text{m s}^{-1}$, the external resistances to motion are $(10{\cdot}5 + 0{\cdot}4v^2)$ N.

Given that the total mass of the cyclist and bike is 66 kg, calculate the power produced by the cyclist. **4**

4. A train travels from Glasgow to Stirling. It starts from rest and accelerates uniformly for the first 9 km of its journey. It then travels for 46·8 km at a uniform velocity, before decelerating uniformly to rest in 7·2 km. The total journey time is 33 minutes.

(a) Sketch a velocity–time graph with appropriate units to represent this journey. **2**

(b) Calculate, in km h^{-1}, the maximum speed reached by the train. **4**

(c) State **one** assumption you have made in answering this question. **1**

5. A particle is projected vertically upwards at a speed of $14{\cdot}8\,\text{m s}^{-1}$ from a point O. Two seconds later a second particle is projected vertically upwards from O at a speed of $5{\cdot}2\,\text{m s}^{-1}$.

(a) Calculate the value of t when the two particles collide assuming that the only force acting is that due to gravity. **4**

(b) Determine the distance the particles are above O when they collide. **1**

MARKS

6. An object moves horizontally along the x-axis with simple harmonic motion about a point O. The period of the oscillation is 12 seconds. It is released from its extreme position A, a distance of 3 metres from O.

Find the first time the particle will be a distance of 4 metres from A. **4**

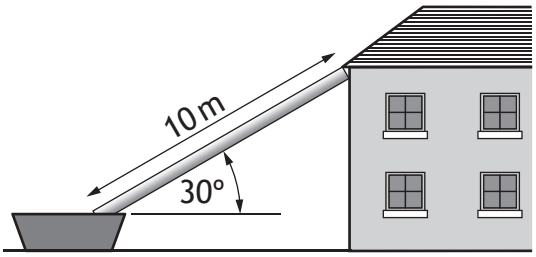
7. Calculate the gradient of the tangent to the curve $xy^2 - 4xy = 5$ at the point (1,5). **4**

8. A stone of mass 0·25 kg moves along a smooth horizontal surface. It passes a fixed point O with a velocity of $3\mathbf{i}$ m s^{-1}. When it is at a distance x metres from O, a force of magnitude $(2\sin x + 5)\mathbf{i}$ N acts on it along the line of movement.

(a) Calculate the work done by the force in moving the stone 3 metres beyond O. **2**

(b) Find the speed of the stone at this point. **2**

9. A house is being re-roofed. The old tiles slide down a rough plastic chute into a skip at the side of the house. The chute is 10 metres long and inclined at an angle of 30° to the horizontal as shown.



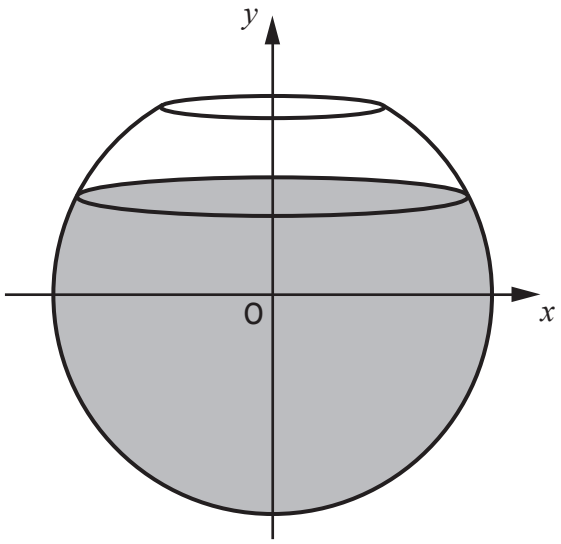
A tile of mass m kg is given an initial speed of 2 m s^{-1} at the top of the chute. The coefficient of friction between the tile and the chute is $\frac{1}{2\sqrt{3}}$.

Show that the speed of the tile at the bottom of the chute is $\sqrt{53}$ m s^{-1}. **5**

10. Find the exact value of $\int_2^7 \frac{x}{\sqrt{x+2}}\,dx$ using the substitution $u = x + 2$. **5**

MARKS

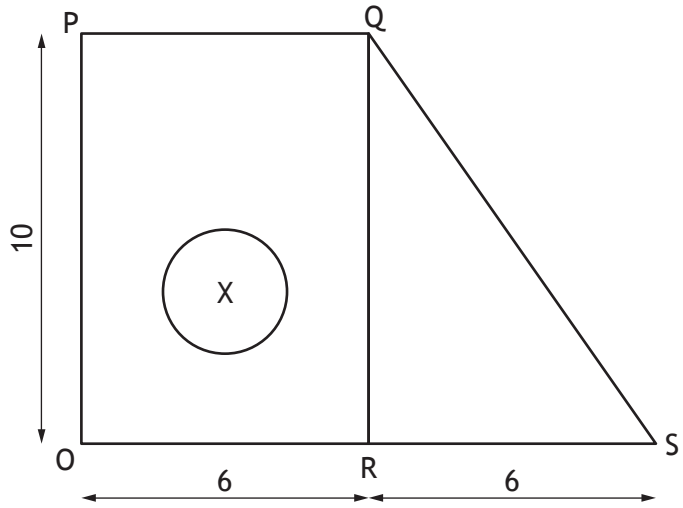
11. A flower vase is modelled by rotating part of the curve $x = \sqrt{100 - y^2}$ through 2π radians about the y-axis as shown in the diagram.



(a) Find the volume of water needed to fill the vase to a depth of 16 cm. **4**

(b) State **one** improvement that could be made to the design **1**

12.



A uniform lamina is made from a rectangle OPQR and a right-angled triangle RQS. A circular hole of radius 1 cm is removed as shown in the diagram. The centre X of the circular hole is 3 cm from both OS and OP.

(a) Taking O as the origin, find the coordinates of the centre of mass of the lamina. **5**

(b) When the lamina is suspended from a point T on OP, it hangs in equilibrium with OS vertical. Give the length of OT. **1**

MARKS

13. The distance of the Earth from the Sun is $1{\cdot}50 \times 10^{11}$ metres.

The distance of Venus from the Sun is $1{\cdot}08 \times 10^{11}$ metres.

Calculate the period of rotation of Venus around the Sun, giving your answer in Earth years.

State **one** assumption you have made when calculating your answer. **6**

14. Three vessels A, B and C are being tracked by coastguards at half-hour intervals. With distances measured in kilometres and speeds in kilometres per hour, they have the following displacement and velocity vectors:

Vessel	A	B	C
Time	10:00	10:30	11:00
Position	$2\mathbf{i}+7\mathbf{j}$	$6\mathbf{i}+9\mathbf{j}$	$12\mathbf{i}+9\mathbf{j}$
Velocity	$4\mathbf{i}+5\mathbf{j}$	$3\mathbf{i}+4\mathbf{j}$	$2\mathbf{i}+6\mathbf{j}$

(a) Show that if A and C continue without changing course they will collide.

Find the time and position of the collision. **5**

At the instant of the collision, vessel B changes course and then proceeds directly to the scene of the collision at its original speed.

(b) Find the time, to the nearest minute, at which vessel B will arrive at the scene of the collision and state the bearing of its course to this point. **5**

15. A golfer hits a ball from the point O with velocity $(P\mathbf{i}+Q\mathbf{j})$ m s^{-1}. The ball first hits the ground a distance of 50 metres from O in the horizontal plane.

(a) Show that $PQ=25g$. **4**

(b) Given that the ball passes through $45\mathbf{i}+1{\cdot}6\mathbf{j}$

(i) Calculate P. **4**

(ii) Calculate the initial angle of projection to the horizontal. **2**

MARKS

16. The movement of a door-closer on a hinged door is modelled by the differential equation $\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = 0$.

(a) Find the solution $y = f(t)$ to this differential equation, given that $y = 1$ and $\frac{dy}{dt} = 2$ when $t = 0$. **6**

(b) State which type of damping is described by the motion and give a reason for your answer. **2**

17. A car of mass M kg is travelling with a speed of v m s^{-1} round a circular bend of radius 40 metres on a road banked at 30° to the horizontal. The coefficient of friction between the car tyres and the road surface is μ.

(a) Show that the square of the maximum speed the car can travel without slipping is given by

$$v^2 = \frac{392(1+\sqrt{3}\mu)}{\sqrt{3}-\mu}$$

5

The minimum speed that the car can travel round the bend without slipping is u m s^{-1}.

(b) Given that $v = 3u$, calculate the coefficient of friction between the car tyres and the road. **6**

[END OF SPECIMEN QUESTION PAPER]

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Mathematics of Mechanics

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

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Mathematics of Mechanics

Marking Instructions

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General Marking Principles for Advanced Higher Mathematics of Mechanics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(d) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(e) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(f) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.

(g) Unless specifically mentioned in the Detailed Marking Instructions, do not penalise:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in solutions
- repeated errors within a question

(h) Any rounded answer should be accurate to three significant figures (or one decimal place for angles given in degrees) unless otherwise stated. If an answer differs due to rounding or prior rounding the candidate may be penalised. Only penalise one mark in any question.

Definitions of Mathematics-specific command words used in this Specimen Question Paper

Determine: determine an answer from given facts, figures, or information.

Find: obtain an answer showing relevant stages of working.

Hence: use the previous answer to proceed.

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

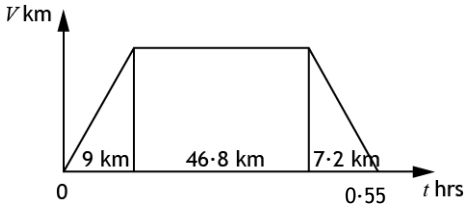
Show that: use mathematics to show that a statement or result is correct (without the formality of proof) — all steps, including the required conclusion, must be shown.

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features.

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question	Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
1	Ans: 0·5 m s^{-1}	3	
	●^{1} conservation of linear momentum		$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ●^{1} $18\begin{pmatrix}0\\-1\cdot1\end{pmatrix}+20\begin{pmatrix}0\\0\end{pmatrix}=18v_1+20\begin{pmatrix}0\cdot36\\-0\cdot72\end{pmatrix}$
	●^{2} calculate v_1		●^{2} $v_1=\begin{pmatrix}-0\cdot4\\-0\cdot3\end{pmatrix}$
	●^{3} calculate speed as the magnitude of velocity		●^{3} $\sqrt{(-0\cdot4)^2+(-0\cdot3)^2}$ = 0·5 m s^{-1}
Notes:			
2	Ans: $e^{x^2}(2x\cos x-\sin x)$	3	
	●^{1} know and use chain rule		●^{1} $2xe^{x^2}$
	●^{2} know and start to use product rule		●^{2} $2xe^{x^2}\cos x+\ldots$
	●^{3} complete process		●^{3} $2xe^{x^2}\cos x+e^{x^2}(-\sin x)$
Notes:			
3	Ans: 136 W	4	
	●^{1} calculate velocity of cyclist		●^{1} $v=\frac{2400}{12\times60}=\frac{10}{3}$ m s^{-1}
	●^{2} find force produced by cyclist to overcome gravity		●^{2} $F=66g\sin\theta+\ldots.$
	●^{3} find total force produced by cyclist		●^{3} $F=66g\sin\theta+(10\cdot5+0\cdot4\left(\frac{10}{3}\right)^2)=40\cdot8$
	●^{4} use $P=Fv$ to find power		●^{4} $P=Fv: P=40\cdot8\times\frac{10}{3}=136$ W
Note: ●^{3} and ●^{4} are still available to candidates who miss ●^{2}.			

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
4	a		Ans:  ●[1] sketch velocity—time graph ●[2] correct annotations	2	●[1] v-t graph with 0 at the origin and 0·55 at the RHS of the t-axis (t hrs). ●[2] labelling of areas under the graph (9 km, 46·8 km and 7·2 km)
4	b		Ans: $v = 144$ km h^{-1} ●[3] set up two appropriate equations of motion ●[4] elimination of one variable ●[5] algebraic manipulation to find v ●[6] calculate maximum speed	4	●[3] $\frac{1}{2}t_1v = 9$ and $(t_2 - t_1)v = 46\cdot8$ or $(t_2 - t_1)v = 46\cdot8$ and $\frac{1}{2}(0\cdot55 - t_2)v = 7\cdot2$ ●[4] either $t_1v = 18$ and $t_2v - t_1v = 46\cdot8$ $\Rightarrow t_2v = 64\cdot8$ or $t_2v - t_1v = 46\cdot8$ and $0\cdot55v - t_2v = 14\cdot4$ $\Rightarrow 0\cdot55v - t_1v = 61\cdot2$ ●[5] either $\frac{1}{2}(0\cdot55 - t_2)v = 7\cdot2 \Rightarrow 0\cdot55v - 64\cdot8 = 14\cdot4$ or $\frac{1}{2}t_1v = 9 \Rightarrow t_1v = 18 \Rightarrow 0\cdot55v - 18 = 61\cdot2$ ●[6] v = 144 km h^{-1}
4	c		Ans: one correct assumption ●[7] correct assumption	1	●[7] train is treated as a particle

Question	Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
Notes:			
1	For ●[1] accept (t mins) with 33 labelled.		
2	For ●[2] accept use of labels for unknown velocity and times at which train is travelling at maximum speed (for example, v, t_1 and t_2).		
3	●[7] is not available if a candidate gives more than one assumption.		
5 a	Ans: $t = 3$	4	
	●[1] consider motion of first particle under constant acceleration		●[1] $s = ut + \frac{1}{2}at^2$ $\Rightarrow s = 14 \cdot 8t - 4 \cdot 9t^2$
	●[2] consider motion of second particle under constant acceleration and substitute time relative to first particle		●[2] $s = ut + \frac{1}{2}at^2$ $\Rightarrow s = 5 \cdot 2(t-2) - 4 \cdot 9(t-2)^2$
	●[3] equate displacements		●[3] $14 \cdot 8t - 4 \cdot 9t^2 = 5 \cdot 2(t-2) - 4 \cdot 9(t-2)^2$ $14 \cdot 8t - 4 \cdot 9t^2 = 5 \cdot 2t - 10 \cdot 4 - 4 \cdot 9t^2 + 19 \cdot 6t - 19 \cdot 6$ $14 \cdot 8t = 24 \cdot 8t - 30$
	●[4] calculate t		●[4] $t = 3$
5 b	Ans: s = 0·3 m (or equivalent)	1	
	●[5] calculate distance above O		●[5] $s = 14 \cdot 8 \times 3 - 4 \cdot 9 \times 3^2 = 0 \cdot 3\text{m}$
Notes:			
6	Ans: $t = 3 \cdot 65\,\text{s}$	4	
	●[1] find the period of the function		●[1] $\omega = \frac{2\pi}{T}$, $\omega = \frac{\pi}{6}$
	●[2] correct formula and begin to substitute correctly		●[2] $3\cos\frac{\pi}{6}t = \ldots$
	●[3] complete substitution and begin to solve		●[3] $3\cos\frac{\pi}{6}t = -1$ $\cos\frac{\pi}{6}t = \frac{-1}{3}$
	●[4] use radians to find a value and find value for t		●[4] $\frac{\pi}{6}t = 1 \cdot 91$, $t = 3 \cdot 65\,\text{s}$
Note: Accept correct solution arrived at using alternative formula $x = a\sin(\omega t + \alpha)$.			

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
7			Ans: $-\frac{5}{6}$ ●[1] start to differentiate ●[2] complete differentiation ●[3] expression for $\frac{dy}{dx}$ ●[4] value of gradient	4	●[1] $y^2+2xy\frac{dy}{dx}+\ldots$ or $\ldots-4y-4x\frac{dy}{dx}$ ●[2] $y^2+2xy\frac{dy}{dx}-4y-4x\frac{dy}{dx}=0$ ●[3] $\frac{dy}{dx}=\frac{4y-y^2}{2xy-4x}$ or equivalent ●[4] $\frac{dy}{dx}=\frac{4\times5-5^2}{2\times1\times5-4\times1}=-\frac{5}{6}$
Notes:					
8	**a**		Ans: 19·0 J ●[1] work done = $\int_a^b f(x)dx$ with substitution ●[2] integration and answer	2	●[1] $\int_0^3(2\sin x+5)dx$ ●[2] $[-2\cos x+5x]_0^3=19\cdot0\,\text{J}$
8	**b**		Ans: 12·7 m s^{-1} ●[3] work done = change in energy ●[4] calculation of speed	2	●[3] increase in $E_K=\frac{1}{2}m(v^2-u^2)$ ●[4] $19\cdot0=\frac{1}{2}(0\cdot25)(v^2-3^2)$ $v=12\cdot7\,\text{m s}^{-1}$
			Alternative solution using Newton's second law This solution gives the answer for (b) before (a) but is acceptable as long as the whole solution is in logical order and the answers for each part clearly identified.		
8	**b**		Ans: 12·7 m s^{-1} ●[1] use of $F = ma$ with appropriate substitution ●[2] integration and completion	2	●[1] $F=ma:\ 2\sin x+5=0\cdot25v\frac{dv}{dx}$ ●[2] $\int0\cdot25v\,dv=\int(2\sin x+5)dx$ $\frac{1}{8}v^2=5x-2\cos x+c$ $x=0,v=3:c=\frac{25}{8}\Rightarrow v^2=40x-16\cos x+25$ $x=3:v^2=120-16\cos(3)+25\Rightarrow v=12\cdot7\,\text{m s}^{-1}$

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
8	**a**		Ans: 19·0 J ●[3] increase in KE = work done ●[4] calculate work done	2	●[3] Work = $\frac{1}{2}m(v^2-u^2)$ ●[4] $=\frac{1}{2}(0\cdot25)(12\cdot7^2-3^2)=19\cdot0\,\text{J}$
Note: For 8a accept 19 J.					
9			Ans: proof ●[1] form equation using resultant force ●[2] find normal reaction force ●[3] substitute all values into equation ●[4] find acceleration ●[5] use STUVA equation and complete	5	●[1] $mg\sin\theta-\mu N=ma$ ●[2] $N=mg\cos\theta$ ●[3] $\frac{mg}{2}-\frac{1}{2\sqrt{3}}\times\frac{\sqrt{3}mg}{2}=ma$ ●[4] $a=\frac{g}{4}$ ●[5] $v^2=2^2+2\times\frac{g}{4}\times10$, $v^2=53$ and complete
9			**Alternative 1**		
			●[1] find total energy at top ●[2] find total energy at bottom and hence the change in energy ●[3] use frictional force to find work done ●[4] use work-energy principle ●[5] complete proof	5	●[1] $\frac{1}{2}m\times2^2+mg\times10\sin30=2m+5mg$ ●[2] loss in $E_k=2m+5mg-\frac{1}{2}mv^2$ ●[3] $\mu N\times d=\left(\frac{1}{2\sqrt{3}}\times mg\cos30\right)\times10=\frac{5}{2}mg$ ●[4] $\frac{5mg}{2}=\frac{1}{2}m\left(v^2-4\right)$ ●[5] $v^2=53$ and complete
9			**Alternative 2**		
			●[1] use $W=Fd$ with correct resultant force ●[2] substitute into $W=Fd$ and simplify expression for work done ●[3] find expression for change in kinetic energy	5	●[1] $W=\left(mg\sin\theta-\mu mg\cos\theta\right)d$ ●[2] $10mg\left(\sin30-\frac{1}{2\sqrt{3}}\cos30\right)=\frac{5mg}{2}$ ●[3] $\frac{1}{2}mv^2-\frac{1}{2}m\times2^2=\frac{1}{2}m\left(v^2-4\right)$

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
			●[4] use work–energy principle ●[5] complete proof		●[4] $\frac{5mg}{2}=\frac{1}{2}m\left(v^2-4\right)$ ●[5] $v^2=53$ and complete
Notes:					
10			Ans: $=8\frac{2}{3}$ ●[1] change variable and differentiate ●[2] change the limits ●[3] write the integral in terms of u ready to be integrated ●[4] integrate ●[5] obtain correct answer	5	 ●[1] $u=x+2\Rightarrow du=dx$ ●[2] when $x=7$, $u=9$ and when $x=2$, $u=4$ ●[3] $\int_4^9\frac{u-2}{\sqrt{u}}du=\int_4^9\left(u^{1/2}-2u^{-1/2}\right)du$ ●[4] $=\left[\frac{2u^{3/2}}{3}-4u^{1/2}\right]_4^9=6-(-2\frac{2}{3})$ ●[5] $=8\frac{2}{3}$
10			**Alternative solution** A candidate may choose not to change the limits, as illustrated in mark 2. In this case mark as follows:		
			●[1] change variable and differentiate ●[2] write the integral in terms of u ready to be integrated ●[3] integrate ●[4] substitute in equation for x ●[5] obtain correct answer	5	●[1] $u=x+2\Rightarrow du=dx$ ●[2] $\int\frac{u-2}{\sqrt{u}}du=\int(u^{\frac{1}{2}}-2u^{-\frac{1}{2}})du$ ●[3] $=\left[\frac{2u^{\frac{3}{2}}}{3}-4u^{\frac{1}{2}}\right]$ ●[4] $=\left[\frac{2(x+2)^{\frac{3}{2}}}{3}-4(x+2)^{\frac{1}{2}}\right]_2^7$ $=6-(-2\frac{2}{3})$ ●[5] $=8\frac{2}{3}$
Notes:					
11	**a**		Ans: 3750 cm^3 ●[1] state integral to be used to find volume ●[2] interpret diagram to state limits	4	 ●[1] $V=\int\pi x^2dy=\int\pi(100-y^2)dy$ ●[2] $\int_{-10}^{6}\pi(100-y^2)dy$

<table>
<tr><th colspan="3">Question</th><th>Expected response
(Give one mark for each ●)</th><th>Max mark</th><th>Additional guidance
(Illustration of evidence for awarding a mark at each ●)</th></tr>
<tr><td></td><td></td><td></td><td>●[3] complete integration

●[4] calculate integral</td><td></td><td>●[3] $V=\pi[100y-\frac{y^3}{3}]_{-10}^{6}$

●[4] 3750</td></tr>
<tr><td>12</td><td>b</td><td></td><td>Ans: valid improvement

●[5] state improvement</td><td>1</td><td>

●[5] eg vase would need a flat base</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td>12</td><td>a</td><td></td><td>Ans: C(4·73, 4·50)

●[1] state area and position of CoM for one of the regular shapes
●[2] state area and position of CoM for other two regular shapes

●[3] take moments about OP equating to moment of lamina
●[4] find x coordinate

●[5] take moments about OS to find y coordinate</td><td>5</td><td>

●[1] and ●[2]<table><tr><th></th><th>Area</th><th>From OP</th><th>From OS</th></tr><tr><td>OPQR</td><td>60</td><td>3</td><td>5</td></tr><tr><td>QRS</td><td>30</td><td>8</td><td>$\frac{10}{3}$</td></tr><tr><td>Circle</td><td>π</td><td>3</td><td>3</td></tr><tr><td>Lamina</td><td>$90-\pi$</td><td>x</td><td>y</td></tr></table>

●[3] $60\times3+30\times8-\pi\times3=(90-\pi)\times x$

●[4] $x=\frac{420-3\pi}{90-\pi}=4\cdot73$

●[5] $60\times5+30\times\frac{10}{3}-\pi\times3=(90-\pi)\times y$</td></tr>
<tr><td>12</td><td>b</td><td></td><td>Ans: OT = 4·50

●[6] interpret rotation</td><td>1</td><td>

●[6] 4·50</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td>13</td><td></td><td></td><td>Ans: 0·611 years

●[1] state any valid assumption

●[2] calculate angular velocity of Earth</td><td>6</td><td>

●[1] eg orbits are circular, any force created by the rotation of the planets can be ignored
●[2] 1 Earth year = 31536000 seconds, hence
$\omega_E=\frac{2\pi}{60\times60\times24\times365}\approx1\cdot99\times10^{-7}$</td></tr>
</table>

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
			●[3] use inverse square law and centripetal force for either planet ●[4] use method for second planet and calculate constant ●[5] calculate angular velocity of Venus ●[6] calculate period in Earth years		●[3] $m_E r_E \omega_E^{\,2} = \frac{GMm_E}{r_E^{\,2}}$ or $m_V r_V \omega_V^{\,2} = \frac{GMm_V}{r_V^{\,2}}$ ●[4] $GM = r_E^{\,3}\omega_E^{\,2} = (1{\cdot}5\times10^{11})^3 \times \left(\frac{2\pi}{31536000}\right)^2$ $= 1{\cdot}3397\times10^{20}$ ●[5] $\omega_V^{\,2} = \frac{GM}{r_V^{\,3}} = \frac{1{\cdot}3397\times10^{20}}{(1{\cdot}08\times10^{11})^3}$ so $\omega_V = 3{\cdot}2612\times10^{-7}$ ●[6] $T = \frac{2\pi}{3{\cdot}2612\times10^{-7}} = 19266612\text{ s} = 0{\cdot}611$ years
Note: A correct answer using angular velocity in radians per earth year instead receives full credit.					
14	**a**		Ans: 1400, $\begin{pmatrix}18\\27\end{pmatrix}$ ●[1] find position vector of A at 1100 ●[2] express displacement for A and C after t hours ●[3] prove collision ●[4] state time at collision ●[5] state displacement vector at collision	5	●[1] $\begin{pmatrix}2\\7\end{pmatrix}+\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}6\\12\end{pmatrix}$ ●[2] $\begin{pmatrix}6\\12\end{pmatrix}+t\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}6+4t\\12+5t\end{pmatrix}$ and $\begin{pmatrix}12\\9\end{pmatrix}+t\begin{pmatrix}2\\6\end{pmatrix}=\begin{pmatrix}12+2t\\9+6t\end{pmatrix}$ ●[3] $6+4t=12+2t \Rightarrow t=3$ $12+5t+9+6t \Rightarrow t=3$ proving displacements same after 3 hours ●[4] collision: 1400 ●[5] collision: position: $\begin{pmatrix}18\\27\end{pmatrix}$
14	**b**		1451, 020·6° ●[6] state position of B at 1400 ●[7] find distance from B to collision point ●[8] find time to collision point using speed of B	5	●[6] $\begin{pmatrix}6\\9\end{pmatrix}+\frac{7}{2}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}16{\cdot}5\\23\end{pmatrix}$ ●[7] $\lvert BK\rvert = \sqrt{(18-16{\cdot}5)^2+(27-23)^2} = 4{\cdot}272$ km ●[8] Time = $\frac{4{\cdot}272}{5} = 0{\cdot}854$ hrs = 51 mins Time of arrival 1451

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
			●[9] identify method to find bearing ●[10] calculation of bearing		●[9] $\tan^{-1}(\frac{27-23}{18-16\cdot5}) = 69\cdot4°$ ●[10] Bearing 020·6°
			Alternative		
			●[6] state position of B at 1400 ●[7] statement for displacement in terms of bearing ●[8] solve equations for θ ●[9] state bearing ●[10] calculation of time		●[6] $\begin{pmatrix}6\\9\end{pmatrix}+\frac{7}{2}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}16\cdot5\\23\end{pmatrix}$ ●[7] $\begin{pmatrix}16\cdot5\\23\end{pmatrix}+t\begin{pmatrix}5\sin\theta\\5\cos\theta\end{pmatrix}=\begin{pmatrix}18\\27\end{pmatrix}$ $5t\sin\theta=1\cdot5 \quad 5t\cos\theta=4$ ●[8] $\tan\theta=\frac{1\cdot5}{4}\Rightarrow\theta=20\cdot6°$ ●[9] bearing 020·6° ●[10] $t=\frac{4}{5\cos20\cdot6°}=51$ mins time of arrival 1451
15	**a**		Ans: show $PQ=25g$ ●[1] equate the vertical component of the position to zero, or the vertical component of the velocity to zero ●[2] determine an expression for the time of flight ●[3] substitute into relevant equation ●[4] rearrange to the required result	4	●[1] $Qt-\frac{1}{2}gt^2=0$ or $Q-gt=0$ ●[2] $t=\frac{2Q}{g}$ ●[3] $Pt=50$ ●[4] $P\left(\frac{2Q}{g}\right)=50$, $PQ=25g$ *Alternative solution mark as above with only* ●[4] different. $Pt=50$ and $Qt-\frac{1}{2}gt^2=0$ $t=\frac{50}{P}\Rightarrow Q\left(\frac{50}{P}\right)-\frac{1}{2}g\left(\frac{50}{P}\right)^2=0$ ●[4] $\frac{50Q}{P}-\frac{2500g}{2P^2}=0$ $100PQ-2500g=0$ $PQ=25g$

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
15	**b**	**i**	Ans: $P = 26\cdot 25\ \text{m s}^{-1}$	4	
			●[5] set up equations for the horizontal and vertical components of the position		●[5] $Pt = 45$ and $Qt - \frac{1}{2}gt^2 = 1\cdot 6$
			●[6] substitute equation for t into the equation for the vertical component of the velocity		●[6] $t = \dfrac{45}{P} \quad \Rightarrow Q\left(\dfrac{45}{P}\right) - \frac{1}{2}g\left(\dfrac{45}{P}\right)^2 = 1\cdot 6$ $90PQ - 2025g = 3\cdot 2P^2$
			●[7] substitute to get equation for u^2		●[7] $PQ = 25g \quad \Rightarrow 90(25g) - 2025g = 3\cdot 2P^2$ $2250g - 2025g = 3\cdot 2P^2$
			●[8] solve to find value of u		●[8] $3\cdot 2P^2 = 2205$, $P = 26\cdot 25\ \text{m s}^{-1}$
		ii	Ans: $\alpha^\circ = 19\cdot 6^\circ$	2	
			●[9] evaluate the vertical component of the velocity		●[9] $PQ = 25g \ \Rightarrow Q = 9\frac{1}{3}\ \text{m s}^{-1}$ $\tan\alpha^\circ = \dfrac{P}{Q}$
			●[10] calculate the angle		●[10] $\tan\alpha^\circ = \dfrac{9\frac{1}{3}}{26\cdot 25} \quad \Rightarrow \alpha^\circ = 19\cdot 6^\circ$
Notes:					
16	**a**		Ans: $y = e^{4t} - 2te^{4t}$	6	
			●[1] correct auxiliary equation		●[1] $m^2 - 8m + 16 = 0$
			●[2] correct solution of auxiliary equation		●[2] $(m-4)^2 = 0$ $m = 4$
			●[3] statement of general solution/complementary function		●[3] $y = Ae^{4t} + Bte^{4t}$
			●[4] correct evaluation of A		●[4] $1 = A.1 + 0 \Rightarrow A = 1$
			●[5] correct differentiation of general solution		●[5] $\dfrac{dy}{dt} = 4Ae^{4t} + Be^{4t} + 4Bte^{4t}$
			●[6] substitution to obtain B and particular solution		$2 = 4Ae^0 + Be^0 + 4B.0.e^0$ $\Rightarrow 2 = 4.1 + B$ ●[6] $\Rightarrow B = -2$ $y = e^{4t} - 2te^{4t}$

Question			Expected response (Give one mark for each ●)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each ●)
16	**b**		Ans: correct statement	2	
			●[7] type of damping		●[7] critically damped
			●[8] correct explanation		●[8] since there is a repeated root
Notes:					
17	**a**		Ans: show that $v^2 = \frac{392(1+\sqrt{3}\mu)}{\sqrt{3}-\mu}$	5	
			●[1] identify frictional force acts down slope		●[1] frictional force acts down the slope (accept diagram)
			●[2] resolve forces horizontally		●[2] $R\sin 30^\circ + \mu R\cos 30^\circ = \frac{mv^2}{r}$
			●[3] resolve forces vertically		●[3] $R\cos 30^\circ - \mu R\sin 30^\circ = mg$
			●[4] know to solve simultaneously		●[4] $\frac{\frac{1}{2}+\mu\times\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}-\mu\times\frac{1}{2}} = \frac{v^2}{392}$
			●[5] rearrange to required answer		●[5] $v^2 = \frac{392(1+\sqrt{3}\mu)}{\sqrt{3}-\mu}$
17	**b**		Ans: $\mu = 0{\cdot}403$	6	
			●[6] resolve forces given that friction is now acting up the slope		●[6] $R\sin 30^\circ - \mu R\cos 30^\circ = \frac{mu^2}{r}$ $R\cos 30^\circ + \mu R\sin 30^\circ = mg$
			●[7] obtain an equation for u^2		●[7] $u^2 = \frac{392(1-\sqrt{3}\mu)}{\sqrt{3}+\mu}$
			●[8] equate v^2 and u^2 given $v = 3u$		●[8] $\frac{3528(1-\sqrt{3}\mu)}{\sqrt{3}+\mu} = \frac{392(1+\sqrt{3}\mu)}{\sqrt{3}-\mu}$
			●[9] obtain a quadratic		●[9] $8\sqrt{3}\mu^2 - 40\mu + 8\sqrt{3} = 0$
			●[10] use the quadratic formula to obtain two solutions		●[10] $\mu = \frac{40 \pm \sqrt{832}}{16\sqrt{3}}$ $\mu = 0{\cdot}403$ or $\mu = 2{\cdot}48$
			●[11] state required answer with reason		●[11] $\mu = 0{\cdot}403$ since $0 < \mu < 1$.
Notes:					
1			●[1] can be implied by ●[2] and ●[3].		
2			For candidates who resolve parallel and perpendicular to the slope only ●[1] and ●[4] are available.		

[END OF SPECIMEN MARKING INSTRUCTIONS]