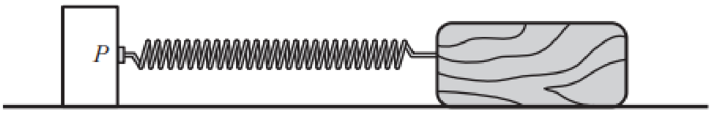
Y	Q	Differential Equations
2024	11	Solve the differential equation $$\frac{dy}{dx} - 2y = 3e^{2x}$$ given that when $x = 0,\ y = 5$. Express y in terms of x. **4**
2024	13	The acceleration of a particle is given by the differential equation $\frac{dv}{dt} = \frac{2v}{1+t}$, where v m s^{-1} is its velocity and t is the time in seconds. Given that the initial velocity of the particle is 2 m s^{-1}, calculate the velocity of the particle after 3 seconds. **4**
2023	14	Solve the differential equation $$9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 0$$ given that when $x = 0,\ y = 6$ and $\frac{dy}{dx} = -3$. **5**
2022	5	An object is launched along the x-axis, from the origin, with an initial velocity of 5 m s^{-1}. The subsequent motion can be modelled by the equation $$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = 0.$$ Find the particular solution for x in terms of t where x is measured in metres and t is measured in seconds. **5**
2022	11	Find the particular solution of the differential equation $$\frac{dy}{dx} - \frac{y}{x} = xe^{2x}$$ given that $y = \frac{3}{2}e^2$ when $x = 1$. Express your answer in the form $y = f(x)$. **6**
2019	5	Find the solution of the second order differential equation $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$ given that $y = 1$ and $\frac{dy}{dx} = 3$, when $x = 0$. **5**

2019	11	A particle of mass 2 kg is projected from an origin along the x-axis with an initial velocity of $5\ \mathrm{m\,s^{-1}}$. A variable force of magnitude $0{\cdot}2v^2$ newtons acts in the opposite direction to the initial motion of the particle, where v is the velocity of the particle in metres per second. Find an expression for the velocity of the particle in terms of its displacement, x metres. Give your answer in the form $v = pe^{qx}$, where p and q are constants. **5**
2018	15	A spring is attached to a fixed point P. The other end is attached to a block of wood on a smooth horizontal surface as shown in the diagram.  The spring is stretched so that the block of wood moves 1·5 metres from its rest position. The block is then projected with a speed of $0{\cdot}5\ \mathrm{m\,s^{-1}}$ towards P at time $t = 0$. The subsequent motion can be modelled by the differential equation $$\frac{d^2x}{dt^2} + 0{\cdot}4\frac{dx}{dt} + 0{\cdot}04x = 0$$ where x metres represents the displacement from the rest position, and t is measured in seconds. (a) Solve this second order differential equation and use the initial conditions given to determine an expression for x in terms of t. **5** (b) Hence calculate how far the block of wood has moved after 2 seconds. **1**
2018	17	A box of mass m kg is set in motion with an initial impulse I. As it moves along the surface it experiences a resistive force proportional to the square of its velocity $v\ \mathrm{m\,s^{-1}}$. By setting up a differential equation, show that the velocity of the box after t seconds can be expressed as $v = \dfrac{mI}{Ikt + m^2}$, where k is a constant and t is measured from the moment of impulse. **5**
2017	16	A body has a velocity $v\ \mathrm{m\,s^{-1}}$ and its motion after t seconds can be modelled as $$\frac{dv}{dt} - \frac{v}{t} = 3$$ Find an expression for its velocity in terms of t, given that the body has a velocity of $5\ \mathrm{m\,s^{-1}}$ after 1 second. **5**

2016	15	A mass of 0·25 kg is attached to a horizontal spring of natural length 1 metre and modulus of elasticity 20 newtons. The spring is stretched and then released. It experiences a resistive force of magnitude $6v$ newtons, where v is the velocity of the mass. (a) Show that the subsequent motion satisfies the second order differential equation $$\frac{d^2x}{dt^2}+24\frac{dx}{dt}+80x=0.$$ (b) Solve this second order differential equation given that the mass is released from rest with an extension in the spring of 0·2 m. (c) Show that the acceleration is equal to zero when $t=\frac{1}{16}\ln 5$ seconds and find the displacement at this time.	 2 6 3
2016 Spec	16	The movement of a door-closer on a hinged door is modelled by the differential equation $\frac{d^2y}{dt^2}-8\frac{dy}{dt}+16y=0$. (a) Find the solution $y=f(t)$ to this differential equation, given that $y=1$ and $\frac{dy}{dt}=2$ when $t=0$. (b) State which type of damping is described by the motion and give a reason for your answer.	 6 2