2015 Applied Mathematics − Mechanics

Advanced Higher

Finalised Marking Instructions

Part One: General Marking Principles for Applied Mathematics – Mechanics – Advanced Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question. If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader/Principal Assessor.

(b) Marking should always be positive ie, marks should be awarded for what is correct and not deducted for errors or omissions.

GENERAL MARKING ADVICE: Applied Mathematics – Mechanics – Advanced Higher

The marking schemes are written to assist in determining the "minimal acceptable answer" rather than listing every possible correct and incorrect answer. The following notes are offered to support Markers in making judgements on candidates' evidence, and apply to marking both end of unit assessments and course assessments.

These principles describe the approach taken when marking Advanced Higher Applied Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values/algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**2**		Acceleration: $s=300 \quad t= \quad u=0 \quad v=20 \quad a=$ $v^2=u^2+2as \qquad v=u+at$ $400=600a \qquad 20=\frac{2}{3}t$ $a=\frac{2}{3}\text{ms}^{-1} \qquad t=30\text{ secs}$ Deceleration: $s= \quad t=15 \quad u=20 \quad v=0 \quad a=$ $v=u+at \qquad v^2=u^2+2as$ $0=20+15a \qquad 0=400-\frac{8}{3}s$ $a=\frac{-4}{3}\text{ms}^{-2} \qquad s=150\text{ metres}$ [Alternatively: Deceleration in half the time: $a=\frac{-4}{3}\text{ms}^{-2}$] Remaining distance at 20ms^{-1} $5000-300-150=4550$ $t=\frac{4550}{20}=227\cdot5$ Total time: $227\cdot5+15+30=272\cdot5$ secs	**5**	**M1**: Use of *stuva* with substitution **E2:** Correct values of *a* and *t* **Graphical approach:** **M1:** Draw v/t graph and correctly interpret data to find acceleration **E2:** Correct values of *a* and *t* **E3:** Deceleration time and distance correct **or** state deceleration directly **M4:** Calculation of time for remaining distance at constant speed **E5:** Correct total time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**3**		$AP^2 = 10^2 + 24^2$ $AP = 26$ Extension in *AP* and *PB* = 6cm $EPE = \frac{\lambda x^2}{2l} = \frac{25(0 \cdot 06)^2}{2(0 \cdot 2)} = 0 \cdot 225$ Total $EPE = 0 \cdot 45$ $KE = \frac{1}{2} mv^2$ $\frac{1}{2}(0 \cdot 02)v^2 = 0 \cdot 45$ $v = 6 \cdot 71 \text{ms}^{-1}$	5	**M1:** Find **extension** in string **M2:** Knowing to use EPE with substitution **E3:** Total EPE = 45J **M4:** Equating kinetic energy and EPE **E5:** Calculate speed
			Note: For $T = \frac{\lambda x}{l} = 7 \cdot 5$ allocate 1 mark For $F = ma$: $2T\cos\theta = ma \Rightarrow a = 692 \cdot 3\text{ms}^{-2}$ allocate 2 marks No further marks could be awarded if candidate starts solution in this way.		
Alternative Solution: Differential Equations					
			$AP^2 = 10^2 + 24^2$ $AP = 26 \Rightarrow$ extension in string =12cm $F = \frac{\lambda x}{l} \Rightarrow ma = \frac{\lambda x}{l} \Rightarrow a = \frac{\lambda x}{lm}$ $v\frac{dv}{dx} = \frac{\lambda}{lm}x$ $\int_0^v v\,dv = \frac{\lambda}{lm}\int_0^{0 \cdot 12} x\,dx$ $\frac{v^2}{2} = \left[\frac{\lambda x^2}{2lm}\right]_0^{0 \cdot 12} \Rightarrow v = 6 \cdot 71\text{ms}^{-1}$	5	**M1:** Find **extension** in string **M2:** Use tension in string to find expression for acceleration **M3:** Set up differential equation **M4:** Separate variables with limits **E5:** Evaluate integral to find speed
Alternative solution: Work/energy principle					
			$AP^2 = 10^2 + 24^2$ $AP = 26 \Rightarrow$ extension in string =12cm Work done = $\int_0^{0 \cdot 12} F\,dx = \int_0^{0 \cdot 12} T\,dx = \int_0^{0 \cdot 12} \frac{\lambda}{l}x\,dx$ $\int_0^{0 \cdot 12} \frac{\lambda}{l}x\,dx = \left[\frac{\lambda x^2}{2l}\right]_0^{0 \cdot 12} = 0 \cdot 45\text{J}$ $\frac{1}{2}mv^2 = 0 \cdot 45$ $v = 6 \cdot 71\text{ms}^{-1}$	5	**M1:** Find **extension** in string **M2:** State work done by string as an integral with limits **E3:** Evaluate integral **M4:** Work/energy principle **E5:** Evaluate speed

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**4**	**a**	↑Equilibrium: $T = 1800g$ $P = Fv \quad or \quad F = \frac{P}{v}$ $P = 1800g \times 4 = 70560 \approx 70 \cdot 6\,\text{kW}$	**2**	**M1:** state tension in cable and use relationship between Power, force and velocity **E2:** Value of power.
A	**4**	**b**	$T - mg = ma$ $T = 1800(g + a)$ $\uparrow = 1800\left(9 \cdot 8 + \frac{4}{7}\right)$ $= 18669N$ $P = Fv \Rightarrow P_{\max} = Fv_{\max}$ $Fv_{\max} 18669 \times 4 = 74676$ $\approx 74 \cdot 7\,\text{kW}$	**2**	**M1:** Use $F=ma$ to find tension under acceleration **E2**: Value of maximum power
A	**4**	**c**	Height = area under s/t graph Height $= \frac{1}{2}(7 \times 4) + (16 \times 4) + \frac{1}{2}(13 \times 4)$ $= 104$ metres	**2**	**M1:** Method of area under s/t graph **E2:** Height of lift

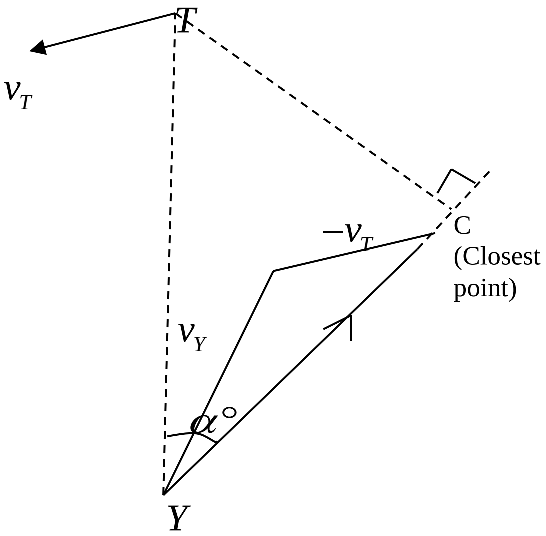
Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**5**	**a**	$\frac{GMm_B}{r^2} = \frac{m_B v_B^2}{r} \Rightarrow GM = rv_B^2$ $\frac{GMm_C}{(2r)^2} = \frac{m_C v_C^2}{2r} \Rightarrow GM = 2rv_c^2$ $\frac{v_B^2}{v_c^2} = 2 \Rightarrow v_B^2 = 2v_c^2$ $v_B = \sqrt{2}v_c$ $v_B = r\omega_B \qquad v_C = 2r\omega_C$ $v_B = \sqrt{2}v_c$ $r\omega_B = \sqrt{2} \times 2r\omega_C$ $\omega_B = 2\sqrt{2}\omega_C$	**4**	**M1**: Use of inverse Square Law of Gravitation for both orbits. **E2**: Equating expressions for *GM* and manipulation for answer **M3:** relationship between linear and angular momentum **E4**: Manipulation to give $\omega_B = 2\sqrt{2}\omega_C$
A	**5**	**b**	$P_B = \frac{2\pi}{\omega_B} = n \Rightarrow \omega_B = \frac{2\pi}{n}$ $\omega_B = 2\sqrt{2}\omega_C$ $\frac{2\pi}{n} = 2\sqrt{2}\omega_C \Rightarrow \omega_C = \frac{\pi}{n\sqrt{2}}$ $P_C = \frac{2\pi}{\omega_C} = \frac{2\pi}{\frac{\pi}{n\sqrt{2}}} = 2\sqrt{2}n$ days	**2**	**M1:** Relationship between period and angular velocity with substitution **E2**: Calculation of period for Casper

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**6**	**a**	At P: $E_P = mgh = 60g \times 4\cos 30°$, $E_K = 0$ At release: $E_P = 0$, $E_K = \frac{1}{2}(60)v^2$ Conservation of energy: $30v^2 = 60g \times 4\cos 30°$ $v = 8\cdot 24ms^{-1}$ Motion in a circle $F = ma = \frac{mv^2}{r}$ $: T - 60g\cos 30° = \frac{60(8\cdot 24)^2}{4}$ $T = 90\sqrt{3}g = 1528N \quad \approx 1530N$	**4**	**M1:** Energy considered with expressions for PE and KE at P **E2:** Equating energy at release and calculating value of *v* **M3:** Motion in a circle with correct force **E4:** Calculation of T
A	**6**	**b**	Motion under gravity: $s = h$ $v^2 = u^2 + 2as$ t $0 = (8\cdot 24\sin 30°)^2 - 2gh$ $u = 8\cdot 24\sin 30°$ $h = 0\cdot 866 = \frac{\sqrt{3}}{2}$ $v = 0$ $a = -9\cdot 8$ **Alternative for final four marks** $\frac{1}{2}mv^2 = mgh \Rightarrow \frac{1}{2}v^2 = gh$ $h = \frac{v^2}{2g} \Rightarrow h = \frac{(8\cdot 24\sin 30)^2}{2g}$ $h = 0\cdot 866\,\mathrm{m}$	**2**	**M1:** Consideration of *stuva* with correct substitution **E2:** Value of *h* **M1:** Conservation of energy **E2:** Expression for *h* **M3+E4:** Value of *h*

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	7		After t hours: $r_F = \begin{pmatrix} 4t \\ 20t \end{pmatrix} \quad r_D = \begin{pmatrix} -3t \\ k-4t \end{pmatrix}$ $\|r_Y - r_T\|^2 = (7t)^2 + (24t-k)^2 = 625t^2 - 48kt + k^2$ $\frac{d}{dt}(\|r_Y - r_T\|^2 = 1250t^2 - 48k$ At min dist: $\frac{d}{dt}(\|r_Y - r_T\|^2 = 0 \Rightarrow k = \frac{1250t}{48} = \frac{625t}{24}$ $\|r_F - r_D\|^2 = 625t^2 - 48 \times \frac{625t^2}{24} + (\frac{625t}{24})^2 = 53t^2$ Min dist = 4·2km $53t^2 = 4\cdot2^2 \Rightarrow t = 0\cdot577$ hours = 35 minutes Closest at 3:35pm Original distance apart: $k = 15$ km	6	**M1:** Interpretation of data to give position vectors after time t **E2:** Both position vectors correct **M3:** Expression for [square of] the distance apart **M4:** Method of differentiation to find minimum distance **E5:** Evaluation of time **E6:** Evaluation of k
Alternative solution for using dot product of vectors:					
			${}_Y v_T = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$ ${}_Y r_T = \begin{pmatrix} 7t \\ 24t-k \end{pmatrix}$ ${}_Y v_T \bullet {}_Y r_T = \begin{pmatrix} 7 \\ 24 \end{pmatrix} \bullet \begin{pmatrix} 7t \\ 24t-k \end{pmatrix} = 49t + 576t - 24k$ $49t + 576t - 24k = 0 \Rightarrow k = \frac{625t}{24}$ ${}_Y r_T = \begin{pmatrix} 7t \\ 24t - \frac{625t}{24} \end{pmatrix} \Rightarrow \|{}_Y r_T\|^2 = 53t^2$ $53t^2 = 4\cdot2^2$ $t = 0\cdot577$ hours = 35 minutes Closest at 3:35pm Original distance apart: $k = 15$ km		**M1:** Find relative velocity vector **M2:** Find relative position vector **M3:** for closest approach dot product of relative position and relative velocity vectors = 0 **E4:** Relationship between k and t **E5:** Evaluation of time **E6:** Evaluation of k

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**7**		**(cont)**		

Alternative Solution 2: Using trigonometry



M1: Interpret information to construct diagram to bring yacht/trawler to rest

M2: Closest when perpendicular – marked on diagram

$${}_Y v_T = v_Y - v_T = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$$

$\tan\alpha = \dfrac{7}{24}$ and $\left|{}_Y v_T\right| = 25$ $YC = 25$

Using $\triangle YTC$ $\tan\alpha = \dfrac{4\cdot 2}{25t} = \dfrac{7}{24}$

E3: Find relative velocity components to find value of α

E4: Establish non-vector detail

$t = 0\cdot 577$ hours = 35 minutes

Closest at 3:35pm

M5: Evaluation of time

Original distance apart: $k = 15$ km

E6: Evaluation of k

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	8	a	$mg - mv^2 = mv\frac{dv}{dx}$ $3g - 0\cdot75v^2 = 3v\frac{dv}{dx}$ $g - 0\cdot25v^2 = v\frac{dv}{dx}$ $\int dx = \int \frac{v}{g - 0\cdot25v^2}\,dv$ $x = -2ln\left\|g - 0\cdot25v^2\right\| + c$ $x = 0, v = 0 \rightarrow c = 2lng$ $\therefore x = -2ln\left\|g - 0\cdot25v^2\right\| + 2lng$ $x = 2ln\left\|\frac{g}{g - 0\cdot25v^2}\right\|$ $v = 5: x = 2ln\left\|\frac{g}{g - 6\cdot25}\right\| = 2\cdot03$ metres **<u>Alternative for marks 3, 4 and 5:</u>** $[x]_0^x = \left[-2ln\left\|g - 0\cdot25v^2\right\|\right]_0^5$ $x = -2ln\left\|g - 6\cdot25\right\| + 2lng$ $x = 2ln\left\|\frac{g}{g - 6\cdot25}\right\|$ $x = 2ln\left\|\frac{g}{g - 6\cdot25}\right\| = 2\cdot03$ metres **Note:** $3g - 0\cdot75v^2 = 3\frac{dv}{dt}$ 1st mark awarded	5	**M1:** Use of $F=ma$ **E2:** Simplification and method of separating variables **E3:** Correct integration **M4:** Substitution to find value of c or use limits **E5:** Substitution for v to give displacement **M3:** use of definite integration with correct limits. **E4:** Simplification of log term **E5:** Evaluation of displacement

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**8**	**b**	Work done $=\int_0^a \boldsymbol{F}.\boldsymbol{v}dt$ $\boldsymbol{a}=2t\,\mathbf{i} \rightarrow \boldsymbol{v}=t^2\,\mathbf{i}+\boldsymbol{c}$ $t=0, v=0 \rightarrow \boldsymbol{v}=t^2\,\mathbf{i}$ $\boldsymbol{F}=m\boldsymbol{a} \rightarrow \boldsymbol{F}=10t\,\mathbf{i}$ Work done $=\int_0^a F.vdt=\int_0^a 10t^3dt=\frac{5a^4}{2}$ Work done by P = change in energy: $mgh-\frac{1}{2}mv^2=3g\left(2\cdot 03\right)-\frac{1}{2}\left(3\right)\left(5^2\right)=22\cdot 2J$ $\int_0^a 10t^3dt=22\cdot 2$ $\rightarrow \left[\frac{5t^4}{2}\right]_0^T=\frac{5t^4}{2}=22\cdot 2$ $a=1\cdot 73$ seconds	**5**	**M1:** Statement for work done by a variable force and integration to find expression for *v*. **M2:** Use of F = ma and expression for work done **M3:** Equivalence of work and change in energy with substitution **E4:** Evaluation of change of energy **E5:** Equating answers and evaluating *a*

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**9**	**a i**	A: $\searrow$ $F = ma$ $0\cdot03g\sin30° - T = 0\cdot03a \quad (i)$ B: $\nearrow$ Equilibrium $R_B = 0\cdot02g\cos30° = 0\cdot170$ $\searrow$ $F = ma$ $0\cdot02g\sin30° + T - 0\cdot5R_B = 0\cdot02a \quad (ii)$ Equating expressions for T: $0\cdot03g\sin30° - 0\cdot03a = 0\cdot02a + 0\cdot085 - 0\cdot02g\sin30°$ $0\cdot05g\sin30° - 0\cdot085 = 0\cdot05a$ $a = 3\cdot2ms^{-2}$ $T = 0\cdot03g\sin30° - 0\cdot03(3\cdot2)$ $T = 0\cdot051N$	**4**	**M1:** Consider *A* and *B* separately with equations for equilibrium and motion **E2:** Correct equations **E3:** Acceleration **E4:** Tension
A	**9**	**a ii**	Motion down slope for 0·25m $v^2 = u^2 + 2as$ $v^2 = 2(3\cdot2)(0\cdot25)$ $v = 1\cdot265\ ms^{-1}$ After string breaks:		**M1:** Use of constant acceleration equations with substitution **E2**: Value of *v*

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**9**	**b**	**A :** $0\cdot 03g\sin 30° = 0\cdot 03a$ $a = 4\cdot 9ms^{-2}$ $s = ut + \frac{1}{2}at^2$ $1\cdot 75 = 1\cdot 265t + 2\cdot 45t^2$ $2\cdot 45t^2 + 1\cdot 265t - 1\cdot 75 = 0$ $t = 0\cdot 626$ **B :** $0\cdot 02g\sin 30° - 0\cdot 5R_B = 0\cdot 02a$ $a = 0\cdot 656\,\text{ms}^{-2}$ $s = ut + \frac{1}{2}at^2$ $2 = 1\cdot 265t + 0\cdot 325t^2$ $0\cdot 325t^2 + 1\cdot 265t - 2 = 0$ $t = 1\cdot 207$ Time interval: $1\cdot 207 - 0\cdot 626 = 0\cdot 581\,\text{secs}$	**4**	**M1:** Consider *A* and *B* with correct distances travelled **E2:** Time for *A* **E3:** Time for *B* **E4:** Time interval

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**10**	**a**	$T_{PS} = \frac{\lambda x_{PS}}{l} = \frac{mgx_{PS}}{l}$ $T_{QS} = \frac{\lambda x_{QS}}{l} = \frac{3mgx_{QS}}{l}$ In equilibrium: $T_{PS} = T_{QS}$ $\frac{mgx_{PS}}{l} = \frac{3mgx_{QS}}{l} \Rightarrow x_{PS} = 3x_{QS}$ $x_{PS} + x_{QS} = l$ $x_{PS} + \frac{1}{3}x_{PS} = l \Rightarrow x_{PS} = \frac{3l}{4}$ $PS = l + \frac{3l}{4} = \frac{7l}{4}$	**4**	**M1:** Use of Hooke's law to state tensions in both springs **M2:** Equilibrium and equating tensions **E3:** Establish relationship between extensions **E4:** State the distance *PS*
A	**10**	**b i**	After further extension: $T_{PS} = \frac{\lambda x_{PS}}{l} = \frac{mg\left(\frac{3l}{4} - x\right)}{l}$ $T_{QS} = \frac{\lambda x_{QS}}{l} = \frac{3mg\left(\frac{l}{4} + x\right)}{l}$ Using $\leftarrow F = ma$ $T_{PS} - T_{QS} = ma$ $\frac{mg\left(\frac{3l}{4} - x\right)}{l} - \frac{3mg\left(\frac{l}{4} + x\right)}{l} = m\ddot{x}$ $\ddot{x} = \frac{-4g}{l}x \Rightarrow$ SHM $\omega^2 = \frac{4g}{l}$	**4**	**M1**: State new tensions in each spring **M2:** use of $F = ma$ **E3:** Correct equation **E4:** Complete prove SHM and state value of ω
A	**10**	**b ii**	$v_{\max} = \omega a = \sqrt{\frac{4g}{l}} \times l = 2\sqrt{gl}$ $\Rightarrow k = 2$	**2**	**M1:** Equation for max velocity with substitution **E2:** state value of *k*

[*END OF SECTION A*]

Section B (Mathematics for Applied Mathematics)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**1**		$y = e^{5x}\tan 2x$ $\dfrac{dy}{dx} = e^{5x}.\dfrac{d}{dx}(\tan 2x) + \tan 2x.\dfrac{d}{dx}(e^{5x})$ $= e^{5x}.2\sec^2 2x + \tan 2x.5e^{5x}$ $= e^{5x}(2\sec^2 2x + 5\tan 2x)$	**3**	1: form of product rule 1: one derivative correct 1: other derivative correct (Factorisation not needed)
B	**2**	**a**	$A^2 = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -10 \\ 2 & -4 \end{pmatrix}$ $\det A^2 = (4\times -4) - (2\times -10) = 4$ Since $\det A^2 \neq 0$, inverse of A^2 exists	**2**	1: Matrix A^2 correct 1: correct reason stated
B	**2**	**b**	$A^2B = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ Inverse of $A^2 = \dfrac{1}{4}\begin{pmatrix} -4 & 10 \\ -2 & 4 \end{pmatrix}$ Pre-multiply by $(A^2)^{-1}$ $IB = \dfrac{1}{4}\begin{pmatrix} -4 & 10 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -11 \\ 0 & -5 \end{pmatrix}$ **<u>ALTERNATIVE SOLUTION</u>** Let $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $A^2B = \begin{pmatrix} 4 & -10 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ $4a - 10c = 4$ $4b - 10d = 6$ $2a - 4c = 2$ $2b - 4d = -2$ Hence, $a = 1$ $b = -11$ $c = 0$ $d = -5$	**3**	1: Statement of inverse A^2 1: multiplying both sides by $(A^2)^{-1}$ 1: matrix B 1: Simultaneous equations 1: Two solutions 1: Remaining two solutions.

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	3		$y=\dfrac{\sin x}{2-\cos x}$ $\dfrac{dy}{dx}=\dfrac{(2-\cos x).\cos x-\sin x(\sin x)}{(2-\cos x)^2}$ $=\dfrac{2\cos x-(\cos^2 x+\sin^2 x)}{(2-\cos x)^2}$ $=\dfrac{2\cos x-1}{(2-\cos x)^2}$ For a S.P., $\dfrac{dy}{dx}=0\Leftrightarrow\dfrac{2\cos x-1}{(2-\cos x)^2}=0$ $\Leftrightarrow 2\cos x-1=0$ $\Leftrightarrow \cos x=\dfrac{1}{2}$ $x=\dfrac{\pi}{3}$ when $x=\dfrac{\pi}{3}$, $y=\dfrac{\sin\frac{\pi}{3}}{\left(2-\cos\frac{\pi}{3}\right)}=\dfrac{\sqrt{3}}{3}$	5	1: form of quotient rule with substitution or product rule 1: derivative 1: Use of $\sin^2 x+\cos^2 x=1$ to simplify expression 1: x coordinate 1: y coordinate
B	4		$\log_a 2+\log_a 4+\log_a 8=6\log_a 2$ $\sum_{r=1}^{100}\log_a 2^r=\log_a 2+\log_a 2^2+\log_a 2^3+...+\log_a 2^{100}$ $=\log_a 2(1+2+3+....+100)$ $=\log_a 2\left(\dfrac{100(101)}{2}\right)$ $= 5050\log_a 2$	4	1: Statement of answer 1: Expansion 1: simplification of indices ***and*** factorising 1: correct answer

<table>
<tr><th colspan="3">Question</th><th>Expected Answer(s)</th><th>Max Mark</th><th>Additional Guidance</th></tr>
<tr><td>B</td><td>5</td><td></td><td>
$$\frac{1}{\cos x}\frac{dy}{dx}+y\tan x=\tan x$$
$$\cos x\times\frac{1}{\cos x}\frac{dy}{dx}+\cos x.y\tan x=\cos x.\tan x$$
$$*\ \frac{dy}{dx}+y\sin x=\sin x$$
Integrating Factor is $e^{\int \sin x dx}$ $= e^{-\cos x}$
$$e^{-\cos x}.\frac{dy}{dx}+e^{-\cos x}.y\sin x=e^{-\cos x}.\sin x$$
$$\frac{d}{dx}\left(y.e^{-\cos x}\right)=e^{-\cos x}.\sin x$$
Integrate both sides,
$$\int\frac{d}{dx}\left(y.e^{-\cos x}\right)dx=\int e^{-\cos x}.\sin x\ dx$$
$$ye^{-\cos x}=e^{-\cos x}+C$$
$$y=1+\frac{C}{e^{-\cos x}}$$
General Solution $y = 1 + Ce^{\cos x}$

<u>ALTERNATIVE SOLUTION</u> – From *
$$\frac{dy}{dx}=\sin x(1-y)$$
$$\int\frac{dy}{1-y}=\int\sin xdx$$
$$-\ell n|1-y|=-\cos x+C$$
$$e^{-\ell n|1-y|}=e^{-\cos x+C}$$
$$\Leftrightarrow\frac{1}{1-y}=Ae^{-\cos x}$$
$$\Leftrightarrow 1-y=Be^{\cos x}$$
$$\Leftrightarrow y=1-Be^{\cos x}$$
</td><td>6</td><td>1: Multiply by cosx,

1: form of I.F.

1: I.F

1: expressing LHS as correct exact differential

1: Integrating RHS

1: Explicit function for y

1: Separate variables

1: Integrate both sides,

1: Take exponential of both sides

1: Algebra of exponentials

1: Explicit function for y</td></tr>
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Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**6**	**a**	$\frac{1}{1-y^2}=\frac{1}{(1+y)(1-y)}=\frac{A}{1+y}+\frac{B}{1-y}$ $1 = A(1-y) + B(1+y)$ $A=\frac{1}{2}$ $B=\frac{1}{2}$ $\frac{1}{1-y^2}=\frac{1}{2}\left(\frac{1}{1+y}+\frac{1}{1-y}\right)$	**3**	1: form of partial fractions 1: constant value A 1: constant value B
B	**6**	**b**	**Substitution integral:** $u=\sqrt{1-x}$ $\frac{du}{dx}=\frac{1}{2}(1-x)^{-\frac{1}{2}}\times -1$ $=\frac{-1}{2\sqrt{1-x}}$ $-2du=\frac{dx}{\sqrt{1-x}}$ **Using** $u=\sqrt{1-x}$ $x=1-u^2$ $\int\frac{dx}{x\sqrt{1-x}}$ $=\int\frac{-2du}{x}$ $=-2\int\frac{du}{1-u^2}$ $=-2\int\frac{1}{2}\left(\frac{1}{1+u}+\frac{1}{1-u}\right)du$ $=-\left(\ell n\lvert 1+u\rvert-\ell n\lvert 1-u\rvert\right)+C$ $=\ell n\left\lvert 1-\sqrt{1-x}\right\rvert-\ell n\left\lvert 1+\sqrt{1-x}\right\rvert+C$ $=\ell n\left\lvert\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right\rvert+C$	**6**	1: correct derivative 1: express x in terms of u 1: replace all terms 1: use of partial fractions 1: integration 1: replace all u terms (do not penalise omission of + C or moduli signs)

[END OF SECTION B**]**

[END OF MARKING INSTRUCTIONS**]**