Detailed Marking Instructions for each question

Question		on	Generic Scheme Illustrative Scheme I	Max Mark			
1.			• ¹ calculate impulse • ¹ $F \times t = -180 \times 1.5 = -270 \text{ Ns}$	3			
			• ² equate impulse to change in momentum $e^{2} -270 = mv - mu = m(v - u)$				
			• ³ calculate change in velocity and final velocity $v = 12 - \frac{270}{70} = 12 - 3\frac{6}{7} = 8\frac{1}{7}(m s^{-1})(8 \cdot 14)$				
No 1 1.	Notes: 1. Direction of motion must be implied in either \bullet^1 or \bullet^2						
Сог	Commonly Observed Responses:						

	Alternative solution					
	• ¹	Use $F = ma$ to find acceleration	• ¹ $-180 = 70a \Longrightarrow a = -\frac{18}{7} \mathrm{m s}^{-2}$			
	• 2	Use equations of motion with substitution	• $v = u + at$ with indication of direction of motion			
	• 3	Correct value for velocity	• ³ $v = 12 - \frac{18}{7}(1 \cdot 5) = 12 - 3\frac{6}{7} \text{ m s}^{-1}$ = $8\frac{1}{7} \text{ ms}^{-1} (8 \cdot 14 \text{ ms}^{-1})$			

Notes:

Commonly Observed Responses:

2.			• ¹ Resolve in <i>x</i> direction	• $^{1} P \cos 30^{\circ} - Q \cos 60^{\circ} - 80 \cos 60^{\circ} = 0$	4	
			• ² Resolve in <i>y</i> direction	• ² $Q\sin 60^\circ + P\sin 30^\circ - 80\sin 60^\circ - 64 = 0$		
			• ³ Algebraic manipulation to find value of <i>P</i>	• ³ $P = 40\sqrt{3} + 32 \approx 101$ N		
			• ⁴ Find value of Q	• ⁴ $Q = 40 + 32\sqrt{3} \approx 95 \cdot 4$ N		
Notes:						
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Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark	
3.		• ¹ calculate the displacement	• $\mathbf{d} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$	3	
		• ² substitute into formula for work done	• ² work done = F •d = $\begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 2 \end{pmatrix}$		
		\bullet ³ calculate the work done	• 3 = 18+9 = 27 J		
Not	es:				
Cor	nmonl	Observed Responses:	(1.5		
1.	Candic	ates did not know how to use sca	alar product and gave answer as a vector $\begin{pmatrix} 18\\9 \end{pmatrix}$		
4.		y = 2x - e		3	
		• ¹ Show understanding of use of product rule	• $\frac{1}{dx} \frac{dy}{dx} = 1\left(\frac{d}{dx}\left(\frac{1}{x}\right)\right) + x\frac{d}{dx}(\ln x)$		
		• ² Correct differentiation	• $\frac{dy}{dx} = 1 + \ln x$		
		• ³ Find equation of tangent	• ³ when $x = e$ $y = e \ln e = e$ and $m = 2$ y - e = 2(x - e)		
Notes: 1. Simplification not required for • ¹ or • ³					
Cor	nmonl	/ Observed Responses:			

Question		on	Generic Scheme		Illustrative Scheme	Max Mark	
5.			• ¹ Substitute values into SHM eqs $v^2 = \omega^2 (a^2 - x^2)$	• ¹	$4 = \omega^{2} \left(a^{2} - 0 \cdot 005^{2} \right) \text{ or }$ $1 = \omega^{2} \left(a^{2} - 0 \cdot 007^{2} \right)$	5	
			• ² Obtain second equation and equate expressions for ω^2	• 2	second equation and $\omega^2 = \frac{4}{a^2 - 0.005^2} = \frac{1}{a^2 - 0.007^2}$		
			• ³ Find value of <i>a</i>	• ³	$a^2 = 5 \cdot 7 \times 10^{-5}$ $a = 7 \cdot 55 \times 10^{-3}$ m		
			$ullet^4$ Find value of $artheta$	•4	$\omega^2 = \frac{4}{5 \cdot 7 \times 10^{-5} - 0 \cdot 005^2}$		
					$\omega = 353 \cdot 6 \operatorname{rads}$		
			• ⁵ Calculate frequency	• ⁵	$\frac{353 \cdot 6}{2\pi} = 56 \cdot 3 \text{oscillations per}$		
Not	es:			1			
Con 1.	Commonly Observed Responses: 1. 5mm and 7mm not converted to metres						

Question		on	Generic Scheme Illustrative Scheme		Max Mark			
6.	(a)		 ¹ Correct shape of graph 		2			
			• ² All annotations correct					
			v u 15 10 3	0 90 100 t				
Not	Notes:							
Cor	nmo	nly (Observed Responses:					
	(b)	(i)	 ¹ Find area under graph and equate 	• ¹ $\frac{1}{2}(10 \times 15) + \frac{1}{2}(u+15) \times 20$ +60 $u + \frac{1}{2}(10 \times u) = 1725$	2			
			• ² Calculate value of u	• ² $u = 20$				
Not	es:							
Cor	nmo	nly (Observed Responses:					
		(ii)	• ¹ One assumption stated	 ¹ Path of aircraft remains in one plane (2 dimensions) Each section is in vertical plane (no wobble) 	1			
Not	es:		1					
Cor 1.	Commonly Observed Responses: 1. The assumption given had to be about <i>the PATH</i> of the aircraft							

Question		Generic Scheme	Illustrative Scheme	Max Mark
7.		 ¹ Find time for maximum speed 	• 1 max speed when $a = 0$ $4 - \sqrt{t} = 0$ $t = 16 \operatorname{secs}$	4
		• ² Integrate to give expression of velocity with evidence of C = 0 if indefinite integration used.	• ² $v = 4t - \frac{2}{3}t^{\frac{3}{2}} + C$ at $t = 0, v = 0 \Longrightarrow C = 0$, $v = 4t - \frac{2}{3}t^{\frac{3}{2}}$	
		 ³ Calculate velocity after 16 seconds 	• ³ $v_{16} = 4(16) - \frac{2}{3}(16)^{\frac{3}{2}} = \frac{64}{3} \text{ ms}^{-1}$	
		 ⁴ Calculate increase in kinetic energy 	• ⁴ change in energy	
			$=\frac{1}{2} \times 9 \times \left(\frac{64}{3}\right)^2 - 0$	
			= 2048J(2050J)	
		Alternative solution for • ^{3 and 4}		
		• ³ obtain an expression for the work done as an integral	• ³ $F = ma$ $F = 36 - 9t^{\frac{1}{2}}$	
			$W = \int F.vdt$	
			$W = \int_{0}^{16} (36 - 9t^{\frac{1}{2}})(4t - \frac{2}{3}t^{\frac{3}{2}})dt$	
		• ⁴ integrate and solve to obtain the work done	• $^{4} W = \int_{0}^{16} (144t - 60t^{\frac{3}{2}} + 6t^{2})dt$	
			$= \left\lfloor 72t^{2} - 24t^{\frac{3}{2}} + 2t^{3} \right\rfloor_{0}$ $= 2048 \operatorname{J} (2050 \operatorname{J})$	
Not	es:			
Cor	nmon	ly Observed Responses:		
1.	• ¹ Coi	mmon error $4 - \sqrt{t} = 0 \Longrightarrow t = 2$		

Question			Generic Scheme	Illustrative Scheme	Max Mark	
8.	8. (a)		• ¹	Show equivalence of denominators	• $(x+1)(x+3)(x-2) = x^3 + 2x^2 - 5x - 6$	3
			• ²	Divide polynomials to obtain a remainder	• ² $x^3 + 2x^2 - 5x - 6 \overline{\smash{\big)}3x^3 + 8x^2 - 11}$ with any remainder stated	
			• 3	Complete proof	• ³ $3 + \frac{2x^2 + 15x + 7}{x^3 + 2x^2 - 5x - 6}$ or	
					$3 + \frac{2x^2 + 15x + 7}{(x+1)(x+3)(x-2)}$	
Not	es:					
Con	nmon	ly O	bsei	rved Responses:		
	(b)		• ¹	Correct form of partial fractions	• ¹ $\frac{A}{(x+1)} + \frac{B}{(x+3)} + \frac{C}{(x-2)}$	4
			• 2	Form equation	• ² $2x^2 + 15x + 7 = A(x+3)(x-2)$ + $B(x+1)(x-2) + C(x+1)(x+3)$	
			• 3	Any two constant values	• ³ $x = -1 \Rightarrow A = 1$ and/or $x = -3 \Rightarrow B = -2$ and/or $x = 2 \Rightarrow C = 3$	
			• 4	Remaining value and express in correct form	• 4 3 + $\frac{1}{x+1}$ - $\frac{2}{x+3}$ + $\frac{3}{x-2}$	
Not	es:					
Con	nmon	ly O	bsei	rved Responses:		

Question		on	Generic Scheme	Illustrative Scheme	Max Mark	
9.	(a)		• ¹ Horizontal forces: $F = ma$ with substitution	• ¹ $R\sin 32^\circ + 0.3R\cos 32^\circ = \frac{mv^2}{30}$	6	
			• ² Vertical forces in equilibrium with substitution	• ² $R\cos 32^\circ - 0.3R\sin 32^\circ = mg$		
			• ³ Algebraic manipulations	• $3 \frac{\sin 32^\circ + 0.3\cos 32^\circ}{\cos 32^\circ - 0.3\sin 32^\circ} = \frac{v^2}{30g}$		
			• ⁴ Value of V	• ⁴ $v = 18 \cdot 3 \text{ms}^{-1}$		
			• ⁵ Calculate time for 1 lap	• ⁵ $C = \pi d = \pi \times 60 = 188 \cdot 5m$ $t = \frac{d}{v} = \frac{188 \cdot 5}{18 \cdot 3} = 10 \cdot 3$ seconds		
			 ⁶ Calculate number of laps and round down 	• ⁶ $300 \div 10.3 = 29.1 \Longrightarrow 29$ laps		
			Alternative solution to $ullet^5$ and $ullet$	6		
			 ⁵ Find distance travelled in 5 minutes 	• $5 18 \cdot 3 \times 300 = 5490 \mathrm{m}$	6	
			 ⁶ Calculate number of laps and round down 	• ⁶ 5490 ÷ $60\pi = 29 \cdot 1 \Longrightarrow 29$ laps		
Not	tes:					
Cor 1. 2.	 Commonly Observed Responses: 1. Friction was ignored 2. Many candidates do not understand that motion here is horizontal and choose to treat this as motion on a slope by considering equilibrium perpendicular to the slope and then use acceleration along the slope. 					
	(b)		• ¹ Assumption stated	• ¹ Track of negligible width or cyclist remains 30 metres from the centre of the horizontal circle.	1	
Not	tes:	-	·	·		
Cor	nmon	ly O	bserved Responses:			

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		• ¹ Differentiate x and y with respect to time	• ¹ $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 2 - 10t$	2
			• ² Find $\frac{dy}{dx}$	$\bullet^2 \frac{dy}{dx} = \frac{2 - 10t}{4} \left(= \frac{1 - 5t}{2} \right)$	
Note	es:				
Com	mon	ly Ob	served Responses:		
	(b)	(i)	• ¹ Substitute <i>t</i> = 0 to give the initial velocities horizontally and vertically	• $t = 0: \frac{dy}{dx} = \frac{2}{4} = 0.5$	2
			• ² Find angle	• ² $\tan^{-1}(0.5) = 26 \cdot 6^0 \text{ or } 0.464$	
Note	es:				
Com	mon	ly Ob	served Responses:		
		(ii)	 ¹ Condition for angle of 45⁰ below horizontal 	• ¹ $\frac{dy}{dx} = -1$	2
			• ² Solve for time	• ² $t = \frac{6}{10} = 0.6$ seconds	
Note	25				
Com	mon	ly Ob	served Responses:		
1.	In b	(ii) $\frac{d}{d}$	$\frac{y}{x} = 1$ gives an answer of $t = -0.2$	2 seconds which cannot be explained.	

Question		n Gen	eric Scheme	Illustrative Scheme	Max Mark
11.		 ¹ Find the cu ² State to integra substite ³ State to integra substite ⁴ Find mass 	the area under rve the correct al for $A\overline{x}$ with tution the correct al for $A\overline{y}$ with tution the centre of	• ¹ Shaded area = $\int_{0}^{4} x^{3} dx = 64$ sq units • ² $A\overline{x} = \int_{0}^{4} xy dx = \int_{0}^{4} x^{4} dx = 204 \cdot 8$ $\overline{x} = \frac{204 \cdot 8}{64} = 3 \cdot 2$ • ³ $A\overline{y} = \int_{0}^{4} \frac{1}{2} y^{2} dx = \int_{0}^{4} \frac{x^{6}}{2} dx = 1170 \cdot 3$ $\overline{y} = \frac{1170 \cdot 3}{64} = 18 \cdot 3$ • ⁴ Centre of Mass: $(3 \cdot 2, 18 \cdot 3) \left[\frac{16}{5}, \frac{128}{7} \right]$	4
Note	es:			·	
Com	mon	ly Observed I	Responses:		
1. F	orm	Ila for A y no	t known.		
		Alternati	ive solution for $ullet$	3 and 4	
		•3 Use m but Replac •4 Find tl mass	ethod as A $\frac{x}{x}$ ring x and y he centre of	$A\overline{y} = \int_{0}^{64} y(4-x)dy = \int_{0}^{64} y(4-y)^{\frac{1}{3}}dy$ $= \int_{0}^{64} (4y - y^{\frac{4}{3}})dy = \frac{8192}{7} \Rightarrow \overline{y} = \frac{128}{7}$	

Question		on	Generic Scheme	Illustrative Scheme	Max Mark			
12.	12. (a) • ¹ Create speed/distance triangle and annotate		 ¹ Create speed/distance triangle and annotate 	• 1 true speed = $\frac{1080}{2 \cdot 25} = 480$	3			
				480 10° 450				
				$\left \mathbf{v}_{w}\right ^{2} = 480^{2} + 450^{2} - 2 \times 480 \times 450 \times \cos 10^{\circ}$				
			• ² Calculate wind speed	• ² 86.4 km h ⁻¹				
			• ³ Calculate direction	• ³ $\cos^{-1}\left(\frac{86\cdot 4^2 + 450^2 - 480^2}{2\times 86\cdot 4\times 450}\right) = 105\cdot 2^\circ$				
				So bearing is $025 \cdot 2^{\circ}$				
Not 1. 2.	es: Acce Any a	pt 02 angle	5° or 205·2° must include reference to a	compass direction (from S25·2°W)				
Con 1.	nmor Direc	ly Ol tion	bserved Responses: of wind needed to be refere	nced.				
	(b)	(i)	• ¹ State valid reason	 ¹ wind now acting against direction of travel 	1			
Not	Notes:							
Con	Commonly Observed Responses:							

Q	Question		Generic Scheme Illustrative Scheme N N	Max Mark			
		(ii)	• ¹ Redraw diagram (could be implied by mark 2) • ¹ $G_{115\cdot2^{\circ}} \qquad \alpha^{\circ} \qquad C$ $86\cdot4 \qquad 450$	4			
			• ² Calculate an angle and substitute $P = 54 \cdot 8^{\circ}$				
			• ³ Calculate true speed $v^2 = 450^2 + 86 \cdot 4^2 - 2 \times 450 \times 86 \cdot 4 \times \cos 54 \cdot 8^{\circ}$ true speed = $406 \cdot 38$ km h ⁻¹				
			• ⁴ Calculate difference in travel time $t = \frac{1080}{406\cdot38} = 2\cdot66$ hours = 2 hours 40 mins				
			25 mins longer				
Not 1.	Notes: 1. • ⁴ Markers must work with candidate's appropriate rounding.						
Cor	nmor	nly O	bserved Responses:				

Question		on	Generic Scheme	Illustrative Scheme	Max Mark		
Alte	Alternative solution by VECTORS						
12.	(a)		 ¹ Create vector equation and use to find wind vector 	• 1 $2\frac{1}{4}\begin{pmatrix}450\cos 10^{\circ}\\-450\sin 10^{\circ}\end{pmatrix} + 2\frac{1}{4}\mathbf{v}_{w} = \begin{pmatrix}1080\\0\end{pmatrix}$ $\mathbf{v}_{w} = \begin{pmatrix}36\cdot 84\\78\cdot 14\end{pmatrix}$	3		
			• ² Calculate wind speed	• ² $ \mathbf{v}_{\mathbf{w}} = 86 \cdot 4 \mathrm{km}\mathrm{h}^{-1}$			
			 ³Calculate direction 	• ³ $\tan^{-1}(\frac{78 \cdot 14}{36 \cdot 84}) = 64 \cdot 8^{\circ}$ So wind bearing is 025 · 2°			
Note 1. / 2. /	es: Acce Any a	pt 02 angle	5° must include reference to a	a compass direction (from S19°W)			
Com	Commonly Observed Responses:						
(b) (i) • ¹ State valid reason • ¹			• ¹ State valid reason	• ¹ wind now acting against direction of travel	1		
Note	Notes:						
Com	mor	nly O	bserved Responses:				

Question		ion	Generic Scheme	Illustrative Scheme	Max Mark	
		(ii)	 ¹ Redraw diagram (could be implied by mark 2) 	• ¹ G 115·2° α° C 86·4 450 P	4	
			• ² vector equation to find α°	• ² $\begin{pmatrix} -450\cos\alpha\\ -450\sin\alpha \end{pmatrix} + \begin{pmatrix} 86\cdot 4\sin 25\cdot 2^{\circ}\\ 86\cdot 4\cos 25\cdot 2^{\circ} \end{pmatrix} = \begin{pmatrix} -\nu\\ 0 \end{pmatrix}$ $\alpha = 10\cdot 0^{\circ}$		
			• ³ Substitute in vector equation to find time of flight	• ³ $t \begin{pmatrix} -450\cos 10^{\circ} \\ -450\sin 10^{\circ} \end{pmatrix} + t \begin{pmatrix} 86 \cdot 4\sin 25 \cdot 2^{\circ} \\ 86 \cdot 4\cos 25 \cdot 2^{\circ} \end{pmatrix} = \begin{pmatrix} -1080 \\ 0 \end{pmatrix}$ $t = 2 \cdot 66$ hours = 2hr 40 mins		
			 ⁴ Calculate difference in travel time 	• ⁴ Difference in time of flight 25 minutes		
Not 1.	Notes: 1. Markers must work with candidate's appropriate rounding.					
Cor	nmo	nly Ol	oserved Responses:			

Qı	uestion	Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)	• ¹ state integral to be used to find the volume with substitution	• ¹ $V = \int \pi y^2 dx$ $V = \int \pi e^{\frac{x}{6}} dx$	3
		• ² integrate and state limits	• ² $V = \int_{15}^{30} \pi e^{\frac{x}{6}} dx$ = $\left[6\pi e^{\frac{x}{6}} \right]_{15}^{30}$	
		• ³ substitute in limits and calculate the volume	• ³ $V = 6\pi e^5 - 6\pi e^{2.5}$ (2570cm ³)	
Not 1.	es: • ³ Accept	any numeric answer correct to a	t least 3 significant figures (2570cm ³)	
Con	nmonly O	bserved Responses:		
	(b)	 ⁴ state volume equated to half original volume 	• $6\pi e^{\frac{a}{6}} - 6\pi e^{2\cdot 5} = 1285$	3
		● ⁵ solve equation	• ⁵ $e^{\frac{a}{6}} = 80 \cdot 3$ $a = 26 \cdot 3$	
		• ⁶ interpret required solution	• 6 26.3-15=11.3cm Hence line should be positioned 10.1cm up the side of the bowl.	
Not	es:	1	1	<u> </u>
Con	nmonly O	bserved Responses:		

Question		on	Generic Scheme	Illustrative Scheme	Max Mark
14.			• ¹ Consider force <i>P</i> parallel to the plane	• ¹ $P = W \sin \theta - \mu R_1$	7
			• ² Equilibrium perp to slope and substitution	• ² $R_1 = W \cos \theta$ $P = W \sin \theta - \mu (W \cos \theta)$	
			• ³ Consider horizontal force <i>Q</i> along and perp to plane and equation for equilibrium.	• ³ $Q\cos\theta + \mu R_2 = W\sin\theta$ $R_2 = W\cos\theta + Q\sin\theta$ $Q\cos\theta = W\sin\theta - \mu(W\cos\theta + Q\sin\theta)$	
			$ullet^4$ Rearrange for μ	• ⁴ $\mu = \frac{W\sin\theta - Q\cos\theta}{W\cos\theta + Q\sin\theta}$ or $\mu = \frac{W\sin\theta - P}{W\cos\theta}$	
			• ⁵ Equate expressions	• ⁵ $\frac{W\sin\theta - Q\cos\theta}{W\cos\theta + Q\sin\theta} = \frac{W\sin\theta - P}{W\cos\theta}$	
			• ⁶ Simplify algebra	• ⁶ $W^2 \sin \theta \cos \theta - QW \cos^2 \theta =$ $W^2 \sin \theta \cos \theta - PW \cos \theta + QW \sin^2 \theta - PQ \sin \theta$	
			• ⁷ Use $\cos^2 \theta + \sin^2 \theta = 1$ to complete proof	• ⁷ P(Wcos θ + Q sin θ) = $QW(sin^2 \theta + cos^2 \theta)$ $P = \frac{QW}{Q sin \theta + Wcos \theta}$	
Not	es:				L

1. If candidates have block slipping up the slope, they cannot gain •¹ but all other marks are available

Commonly Observed Responses:

Qı	Question		Generic Scheme	Illustrative Scheme	Max Mark
15.	(a)		 ¹ Establish forces involved 	• $ma = -6v - \frac{\lambda}{l}x$ or $-6v - 20x$	2
			• ² substitute values and re-arrange	• ² $0 \cdot 25 \frac{d^2 x}{dt^2} = -6 \frac{dx}{dt} - 20x$	
				$\frac{d^2x}{dt^2} + 24\frac{dx}{dt} + 80x = 0$	
Note	Notes:				
Con	Commonly Observed Responses:				
	(b)		• ¹ set up auxiliary equation	• $^{1}m^{2}+24m+80=0$	6
			• ² solve quadratic equation	• ${}^{2}(m+20)(m+4) = 0 \Longrightarrow m = -4 \text{ or } m = -20$	
			• ³ general solution	• $^{3}x = Ae^{-4t} + Be^{-20t}$	
			• ⁴ initial condition $x = 0 \cdot 2$ when $t = 0$	• ⁴ $0 \cdot 2 = Ae^0 + Be^0 \Longrightarrow A + B = 0 \cdot 2$	
			 ⁵ differentiate to use initial condition for velocity 	• ${}^{5}\frac{dx}{dt} = -4Ae^{-4t} - 20Be^{-20t} \implies -4A - 20B = 0$	
			• ⁶ solve for <i>A</i> and <i>B</i> , particular solution	• $A = 0.25, B = -0.05$ $x = 0.25e^{-4t} - 0.05e^{-20t}$	
Note	es:				
Com	nmon	ly O	bserved Responses:		

Qı	uestion	Generic Scheme	Illustrative Scheme	Max Mark			
	(c)	 ¹ differentiate to obtain an expression for acceleration 	• $\overset{"}{x} = 4e^{-4t} - 20e^{-20t} = 0$	3			
		• ² solve for t	• ² $e^{-4t} = 5e^{-20t}$ -4t = ln 5 - 20t $t = \frac{1}{16} \ln 5$				
		• ³ substitute for t to give displacement, <i>x</i>	• ³ $x = \frac{1}{4}e^{-0.25\ln 5} - \frac{1}{20}e^{-1.25\ln 5} = 0.160 \mathrm{m}$				
Not	es:						
Con	nmonly	Observed Responses:					
16	(a)	• ¹ state the horizontal and vertical equations of motion	• ¹ $x = Vt \cos \theta$ $y = Vt \sin \theta - \frac{1}{2}gt^2$	3			
		• ² rearrange for <i>t</i> and start substitution	• ² $t = \frac{x}{V\cos\theta}$ $y = \left(V\sin\theta \times \frac{x}{V\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2$				
• ³ obtain required equation $y = x \tan \theta - \frac{gx^2}{2V^2 \cos^2 \theta}$ $y = x \tan \theta - \frac{gx^2}{2V^2} (1 + \tan^2 \theta)$							
Not 1.	Notes: 1. • ¹ can be implied by • ² and • ² can be implied by • ³						
Con	Commonly Observed Responses:						

Question		on	Generic Scheme	Illustrative Scheme	Max Mark
Alt	Alternative solution for • ¹ using calculus				
			 ¹ state horizontal and vertical equations of motion using calculus 	$\begin{aligned} \frac{d^2x}{dt^2} &= 0\\ \frac{dx}{dt} &= c \text{ when } t = 0 \frac{dx}{dt} = V \cos \theta \ c = V \cos \theta\\ x &= \int (v \cos \theta) dt = (V \cos \theta) t + c_2 \text{ when } t = 0 \ x = 0 \ c_2 = 0\\ x &= (V \cos \theta) t\\ \frac{d^2y}{dt^2} &= -g\\ \frac{dx}{dt} &= \int (V \sin \theta) dt = -gt + c_3 \text{ when } t = 0 \frac{dy}{dt} = V \sin \theta c_3 = V \sin \theta\\ y &= \int (V \sin \theta - gt) dt\\ y &= (V \sin \theta) t - \frac{1}{2} gt^2 + c_4 \text{ when } t = 0 \ y = 0 \ c_4 = 0\\ y &= (V \sin \theta) t - \frac{1}{2} gt^2 \end{aligned}$	3
Alt	erna	tive	solution for \bullet^1	using equations of motion	
			 ¹ state horizontal and vertical equations of motion using equations of motion 	$x = V \cos \theta \times t$ $s = ut + \frac{1}{2}at^{2}$ $y = (V \sin \theta)t - \frac{1}{2}gt^{2}$	3

Question		Generic Scheme	Illustrative Scheme	Max Mark		
(b))	 ⁴ Use equation from part (a) to obtain two equations for h 	• ⁴ $h = 4h \tan \theta - \frac{16h^2g}{2V^2} (1 + \tan^2 \theta)$ $h = 5h \tan \theta - \frac{25h^2g}{2V^2} (1 + \tan^2 \theta)$	5		
		 ⁵ equate expressions and some algebraic manipulation 	• ⁵ $\frac{h^2 g}{V^2} (1 + \tan^2 \theta) = \frac{2}{25} (5h \tan \theta - h)$ $\frac{h^2 g}{V^2} (1 + \tan^2 \theta) = \frac{1}{8} (4h \tan \theta - h)$			
		• ⁶ Eliminate V	• $\frac{6}{25}(5h\tan\theta - h) = \frac{1}{8}(4h\tan\theta - h)$			
		• ⁷ simplify and eliminate h	• ⁷ $80h \tan \theta - 16h = 100h \tan \theta - 25h$ $20 \tan \theta = 9$			
		 ⁸ find angle of projection 	• ⁸ tan $\theta = \frac{9}{20} \implies \theta = 24 \cdot 2^{\circ} (0.423)$			
Notes:	Notes:					
Commo	only (Observed Responses:				

Question		Generic Scheme	Illustrative Scheme	Max Mark				
(b))	Alternative solution	ternative solution					
		• ⁴ Express as a quadratic and obtain equation.	• ⁴ quadratic passes through the origin so is of the form $y = ax^2 + bx$					
			Passes through the points $ig(4h,hig)$ and					
			(5h,h)					
			$h = 16h^2a + 4hb$					
			$h = 25h^2a + 5hb$					
		 ⁵ solve simultaneous equations to obtain expressions for a and b ⁶ state equation of the 	• ⁵ $5h = 80h^{2}a + 20hb$ $4h = 100h^{2}a + 20hb$ $a = \frac{-1}{20h}$ and $b = \frac{9}{20}$					
		trajectory	$y = \frac{1}{20h}x^{2} + \frac{1}{20}x^{2}$					
		• ⁷ determine gradient when $x = 0$	• ⁷ $\frac{dy}{dx} = \frac{-1}{10h}x + \frac{9}{20}$					
			at $x=0$ $\frac{dy}{dx}=\frac{9}{20}$					
		 ⁸ solve to find angle of projection 	• ⁸ $m = \tan \theta \Longrightarrow \tan \theta = \frac{9}{20}$ $\theta = 24 \cdot 2^{\circ}$					
Notes:	Notes:							
Commo	only (Observed Responses:						

Qı	uesti	on	Generic Scheme	Illustrative Scheme	Max Mark
17.	(a)	(i)	 ¹ Consider energy at two positions 	• ¹ at starting position $E_k = \frac{1}{2}mu^2$ Elsewhere $E_k + E_p = \frac{1}{2}mv^2 + mg(r - r\cos\theta)$	4
			• ² Use conservation of energy	• ² $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(r - r\cos\theta)$	
	• 3		• ³ Find expression for velocity	• ³ $v^2 = u^2 - 2rg(1 - \cos\theta)$	
		(ii)	• ⁴ F = ma radially and show expression for tension	• $T - mg \cos \theta = \frac{mv^2}{r}$ and complete	2
			 ⁵ Interpret condition for full circles 	• ⁵ $T > 0$ when $\theta = 180^{\circ}$ and substitute	
			• ⁶ Find expression for <i>u</i>	• 6 $u > \sqrt{5rg}$	
Not e 1. a	es: accep	ot T	≥ 0 and $u \geq \sqrt{5rg}$		
Com	nmon	ly Oł	oserved Responses: Condition	for complete circle given as $v > 0$	
	(b)		• ⁷ Interpret condition for string going slack and substitute	• ⁷ $T = 0$ and substitute	3
			• ⁸ Find angle	• ⁸ $\cos\theta = -\frac{2}{3}$	
			• ⁹ Find height	• $^{9} h = r - r \cos \theta = \frac{5}{3}r$ $h = r - r \cos \theta = \frac{5}{3}r$	

[END OF MARKING INSTRUCTIONS]