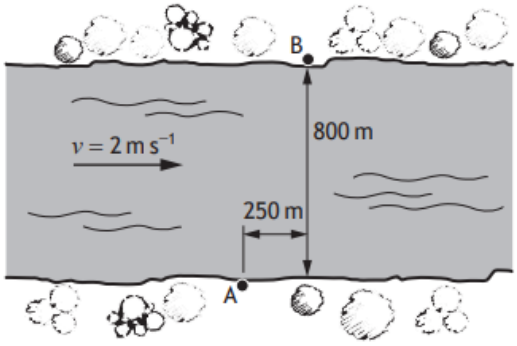
Y	Q	Vector Motion
24	19	A boat Q is moving with a speed of 18 km h^{-1} on a bearing of 270°. To the captain of Q, another boat P appears to be moving at 20 km h^{-1} on a bearing of 015°. (a) Determine: (i) the actual speed of P **3** (ii) the direction in which P is moving. **1** At 12 noon, Q is 12 km from P on a bearing of 290°. Both boats continue on their current paths and at the same speeds. (b) Find the time at which they will pass closest to each other in their subsequent journeys. **5**
23	4	A particle has displacement in metres given by $2t\,\mathbf{i}+3t^2\mathbf{j}-5t^2\,\mathbf{k}$ where t is time in seconds and $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the x, y and z axes respectively. (a) Calculate the velocity of the particle after 3 seconds. **2** (b) Calculate the time when the speed of the particle is 50 m s^{-1}. **2**
23	8	A boat is initially at position $\begin{pmatrix}5\\2\end{pmatrix}$ metres. It is moving with a constant velocity of $\begin{pmatrix}4\\1\end{pmatrix}$ ms^{-1}. A whale is resting at the surface of the water at position $\begin{pmatrix}60\\40\end{pmatrix}$ metres. (a) Determine the closest distance that the boat gets to the whale. **5** (b) State one assumption that you have made. **1**
22	15	A jet takes off from an origin with an initial velocity of $\begin{pmatrix}240\\0\\50\end{pmatrix}$ kmh^{-1}. It then accelerates in a straight line at a constant rate of $\begin{pmatrix}3000\\0\\80\end{pmatrix}$ kmh^{-2} for 12 minutes. (a) (i) Calculate the speed of the jet at this time. **2** (ii) Find the position of the jet at this time. **2** The jet now maintains this speed but on a horizontal course parallel to the x-axis. A wind blows with velocity $\begin{pmatrix}-10\\-50\\0\end{pmatrix}$ kmh^{-1}. (b) (i) Calculate the angle at which the jet is blown off course. **2** (ii) Calculate the horizontal component of the displacement of the jet, 90 minutes after take-off. **3**

19	3	A radio-controlled model boat moves from an origin with velocity $\mathbf{v}(t) = (4\mathbf{i} + (t+1)\mathbf{j})$ ms^{-1}, where t is measured in seconds. The radio signal has a range of 80 metres. Determine whether the boat is still within range of the radio signal after 10 seconds.	4
19	16	A rower is crossing a river that is 800 metres wide. They set off from point A and need to reach point B as quickly as possible. B is 250 metres downstream. They row at a speed of 4 m s^{-1} in still water, and the river current flows at 2 m s^{-1}.  (a) Find at what angle to the bank the rower needs to steer. After rowing for one minute the rower gets tired and immediately reduces speed to 3 m s^{-1}, adjusting steering to maintain the same course. (b) (i) Find how far they are from B at this time. (ii) Calculate the total time it takes the rower to reach point B.	4 3 3
18	11	At 08:00 a port official records Boat A at point (1·2, 1·6) and Boat B at (34·8, 1), where the distances are in kilometres relative to the port as an origin. At 08:06 the official records their points as (6, 3) and (34, 2·5) respectively. (a) Show that their average velocities over this period can be expressed, in km h^{-1}, as $\mathbf{v}_A = 48\mathbf{i} + 14\mathbf{j}$ and $\mathbf{v}_B = -8\mathbf{i} + 15\mathbf{j}$. (b) (i) Assuming that each boat maintains a constant velocity, show that they are on a collision course. (ii) Find the location of the collision.	2 3 1
17	3	The velocity of a particle after t seconds of travel can be expressed as $\mathbf{v} = (3\sin 2t)\mathbf{i} + (\cos 2t - 3)\mathbf{j}$ ms^{-1} where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in horizontal and vertical directions respectively. Find the magnitude of the acceleration of the particle when $t = \frac{\pi}{6}$ seconds.	4

17	14	A fishing boat, A, leaves a harbour with a constant speed of $10\,\text{km h}^{-1}$ on a bearing of 060°. At the same time another fishing boat, B, is 12 km due east of A, moving with a constant speed of $10\sqrt{3}\,\text{km h}^{-1}$ on a bearing of 330°. (a) (i) Describe how the vectors $\mathbf{i}$ and $\mathbf{j}$ should be defined in this situation. (ii) Show that the position of boat A relative to boat B, t hours after A has left the harbour, can be written as ${}_{A}\mathbf{r}_{B} = \left(10\sqrt{3}t - 12\right)\mathbf{i} - 10t\,\mathbf{j}$ kilometres. (b) Find for how long the two boats will be within 7 km of each other. Give your answer to the nearest minute.	2 3 5
16	12	An aircraft flies 1080 km due east from Glasgow to Copenhagen in a time of $2\frac{1}{4}$ hours. The aircraft sets a course on a bearing of 100° and the speed of the aircraft in still air is $450\,\text{km h}^{-1}$. (a) Calculate the magnitude and direction of the wind. (b) (i) Given that the velocity of the wind remains constant, explain why the return journey will take longer. (ii) Calculate how much longer the return journey will take, giving your answer to the nearest minute.	3 1 4
16 (Sp)	14	Three vessels A, B and C are being tracked by coastguards at half-hour intervals. With distances measured in kilometres and speeds in kilometres per hour, they have the following displacement and velocity vectors: (a) Show that if A and C continue without changing course they will collide. Find the time and position of the collision. At the instant of the collision, vessel B changes course and then proceeds directly to the scene of the collision at its original speed. (b) Find the time, to the nearest minute, at which vessel B will arrive at the scene of the collision and state the bearing of its course to this point.	5 5

Vessel	A	B	C
Time	10:00	10:30	11:00
Position	$2\mathbf{i}+7\mathbf{j}$	$6\mathbf{i}+9\mathbf{j}$	$12\mathbf{i}+9\mathbf{j}$
Velocity	$4\mathbf{i}+5\mathbf{j}$	$3\mathbf{i}+4\mathbf{j}$	$2\mathbf{i}+6\mathbf{j}$