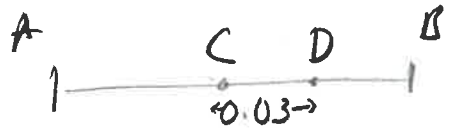
Homework 11 solutions

1) a) $V_{max} = \frac{\pi}{5}$

$\omega = 2\pi/T$

$\omega = 2\pi/0.6$

$\omega = \frac{10\pi}{3}$ ✓

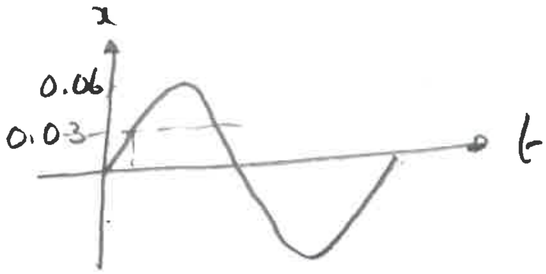


$V_{max} = \omega a$

$\pi/5 = \frac{10\pi}{3} a$ ✓

$\underline{a = 0.06m}$ $\Rightarrow$ $AB = 2 \times 0.06 = \underline{\underline{0.12m}}$ ✓

b) $x = 0.06 \sin \frac{10\pi}{3} t$



$0.06 \sin \frac{10\pi}{3} t = 0.03$ ✓

distance $C \rightarrow D = 0.03$

$\sin \frac{10\pi}{3} t = \frac{1}{2}$

$\frac{10\pi}{3} t = \pi/6$

$\underline{\underline{t = 0.05 secs}}$ ✓

2) $V_{max} = 0.13\,ms^{-1}$ $\qquad$ $V = 0.12,\ x = 0.15$

$\Rightarrow\ wa = 0.13$ $\qquad$ $\Rightarrow\ 0.12^2 = w^2(a^2 - 0.15^2)$

$\underline{w^2a^2 = 0.0169}$ $\qquad$ $0.0144 = w^2a^2 - 0.0225w^2$

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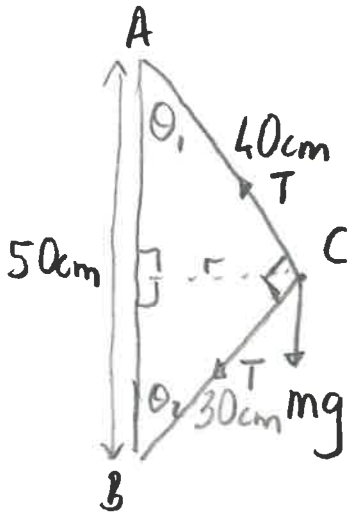
$$w = \frac{1}{3}\ rads^{-1}$$

$T = 2\pi/w$ $\qquad$ $wa = 0.13$

$\underline{T = 6\pi\ secs}$ (18.8 secs) $\qquad$ $\underline{a = 0.39m}$

3)



Tension = T in both AC and BC because it is the same piece of string.

<u>resolve forces vertically</u>

$$T\cos\theta_1 = mg + T\cos\theta_2$$

$\cos\theta_1 = \frac{4}{5}$

$\cos\theta_2 = \frac{3}{5}$

$$T \times \frac{4}{5} = 0.1g + T \times \frac{3}{5}$$

$$\underline{T = 4.9N}$$

<u>resolve forces horizontally</u> $\qquad$ $\Sigma F = ma.$

$$T\sin\theta_1 + T\sin\theta_2 = mw^2r$$

$\sin\theta_1 = \frac{3}{5}$

$\sin\theta_2 = 4/5$

$$4.9 \times \frac{3}{5} + 4.9 \times \frac{4}{5} = 0.1 \times w^2 \times 0.24$$

$$\underline{w = 16.9\ rads^{-1}}$$

also $\sin\theta_1 = \frac{r}{40}$

$\Rightarrow\ r = 40 \times \frac{3}{5} = \underline{24cm}$

4a) $f(x) = \dfrac{1+\sin x}{1+2\sin x}$ 3

$$f'(x) = \frac{\cos x(1+2\sin x) - (1+\sin x).2\cos x}{(1+2\sin x)^2}$$

$$f'(x) = \frac{\cos x + \cancel{2\sin x\cos x} - 2\cos x - \cancel{2\sin x\cos x}}{(1+2\sin x)^2}$$

$$f'(x) = \frac{-\cos x}{(1+2\sin x)^2}$$

b) $g(x) = \ln(1+e^{2x})$

$$g'(x) = \frac{1}{1+e^{2x}} \cdot e^{2x} . 2$$

$$g'(x) = \frac{2e^{2x}}{1+e^{2x}}$$

5a) $v^2 = w^2(a^2 - x^2)$

$\left(\frac{\pi}{\sqrt{3}}\right)^2 = w^2(a^2 - 1^2)$ $\qquad$ $\left(\frac{\pi}{3}\right)^2 = w^2(a^2 - (\sqrt{3})^2)$

$\frac{\pi^2}{3} = w^2(a^2 - 1)$ $\qquad$ $\frac{\pi^2}{9} = w^2(a^2 - 3)$

$$\frac{\frac{\pi^2}{3} = \cancel{w^2}(a^2-1)}{\frac{\pi^2}{9} = \cancel{w^2}(a^2-3)}$$

$$3 = \frac{a^2 - 1}{a^2 - 3}$$

$3a^2 - 9 = a^2 - 1$

$2a^2 = 8$

$a^2 = 4$

$\underline{\underline{a = 2}}$ $\Rightarrow$ amplitude $= \underline{\underline{2m}}$

b) $\frac{\pi^2}{3} = w^2(a^2 - 1)$ $\qquad$ $w = 2\pi/T$

$a = 2 \Rightarrow$ $\quad 3w^2 = \frac{\pi^2}{3}$ $\qquad$ $T = 2\pi/w$

$w^2 = \pi^2/9$ $\qquad$ $\underline{\underline{T = 6 \text{ seconds}}}$

$\Rightarrow w = \pi/3$