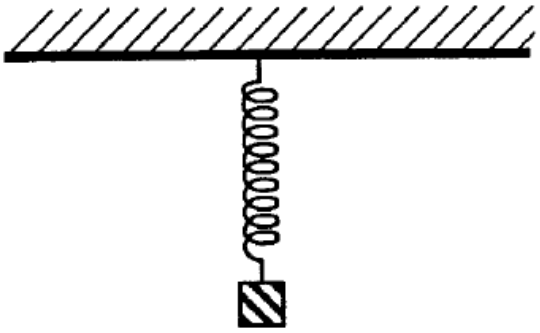
Homework 13

1) A satellite moves in a circular orbit around a planet in the plane of the planet's equator and at a height of 600 kilometres above the surface of the planet. The magnitude of the acceleration due to gravity at the surface of the planet is 11·2 m s^{-2} and the radius of the planet is 8600 kilometres.

Find the time taken by the satellite to complete one orbit. **4**

2) Large springs for shock absorbers are tested at a research laboratory. A body of mass 50kg is suspended from a test spring of natural length 0·80 metres, which has the other end attached to a fixed horizontal surface.



(a) Given that, when in equilibrium, the body extends the spring by 0·14 metres, find the modulus of elasticity of the spring. **2**

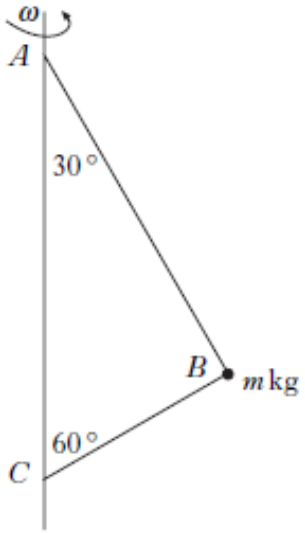
(b) The body is now pulled 0·20 metres vertically down from its equilibrium position and then released from rest. Take y metres as the vertical displacement of the body from its equilibrium position, t seconds after release. Show that, when all resistive forces are ignored,

$$\ddot{y} = -70y$$ **3**

(c) On release what is the speed of the object when it has travelled

0.15 metres from its starting position. **3**

3) A bead of mass m kilograms is attached to a vertical rotating column by two strings, as shown below. String AB is elastic, with natural length L metres and modulus of elasticity $2mg$ newtons. The string is attached to the column at A and to the bead at B. String BC is inextensible and has length L metres. The vertical column is rotating at ω rad s^{-1}, such that the strings AB and BC are taut and remain in a vertical plane. Angles ACB and BAC are $60°$ and $30°$ respectively.



(a) Show that the tension in the string AB is $2(\sqrt{3}-1)mg$ newtons. **4**

(b) Find, in terms of m and g, an expression for the tension in the string BC. **3**

(c) Given that $L = 1$, calculate ω. **4**

4) $$\frac{dy}{dx} = \frac{4x}{4x^2+1}$$

Given that when $x = 0, y = 0$ solve the differential equation and show that

$$x = \frac{\sqrt{e^{2y}-1}}{2}$$ **5**