HOMEWORK 8

Section E (Mechanics 1)

Marks

ONLY candidates doing the course Mathematics 1, 2 and Mechanics 1 should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

Where appropriate, candidates should take the magnitude of the acceleration due to gravity as $9{\cdot}8\ \text{m s}^{-2}$.

E1. A gun is located at one end of a horizontal firing range. A shell is fired down the range at an angle of projection of 45° to the horizontal. After 5 seconds the shell has reached the highest point on its trajectory.

(*a*) Calculate the speed of projection of the shell from the gun. **2**

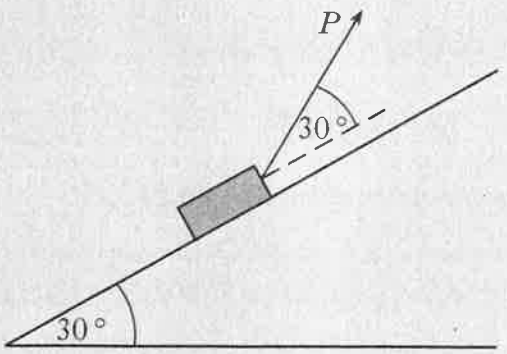
(*b*) Find the horizontal range of the shell. **2**

E2. As a set of traffic lights changes to green, a car accelerates uniformly from rest along a straight horizontal road at $a\ \text{m s}^{-2}$. At the same instant, a lorry, travelling at $U\ \text{m s}^{-1}$ with constant acceleration $\frac{1}{2}a\ \text{m s}^{-2}$, overtakes the car.

(*a*) Show that the car and lorry draw level again after $\frac{4U}{a}$ seconds. **4**

(*b*) Find an expression, in terms of U and a, for the distance travelled by the car when it draws level with the lorry. **1**

E3. A box of mass 2 kg is pulled at a constant speed up a plane inclined at 30° to the horizontal by a force of magnitude P newtons. The force is acting in the direction making an angle of 30° to the line of greatest slope of the plane, as shown. The coefficient of friction between the box and the inclined plane is $\frac{1}{2}$.



(*a*) By resolving perpendicular to the plane, show that the magnitude of the frictional force acting parallel to the inclined plane is

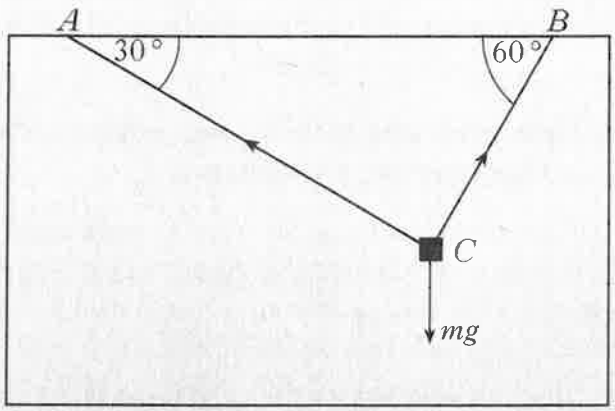
$$\tfrac{1}{4}(2\sqrt{3}g - P) \text{ newtons,}$$

where $g\ \text{m s}^{-2}$ is the magnitude of the acceleration due to gravity. **3**

(*b*) Calculate the magnitude of the force P. **3**

Marks

E4. A box of mass m kilograms is suspended from the ceiling of a lift by means of two light inextensible strings, AC and BC, of differing lengths, as shown below. The strings are attached to the box at the point C and angles CAB and ABC are $30°$ and $60°$ respectively.



The lift has a steady acceleration, vertically upwards, of magnitude a m s^{-2}, during which the tensions in the strings AC and BC are T_1 and T_2 newtons respectively.

(*a*) Show that $T_2 > T_1$. **2**

(*b*) Find an expression for T_2 in terms of m, a and g, where g m s^{-2} is the magnitude of the acceleration due to gravity. **4**

E5. (*a*) John strikes a stationary ice puck A from the origin with initial velocity $10\mathbf{i}$ m s^{-1} where $\mathbf{i}$ is a unit vector. The puck accelerates at $-\frac{2}{5}t\mathbf{i}$ m s^{-2}, where t seconds is the time of travel from the moment the puck is hit. Show that the position of the puck at the time t is

$$\mathbf{r}_A(t) = \tfrac{1}{15}t(150 - t^2)\mathbf{i}.$$ **2**

(*b*) At the same time as John strikes puck A, Julia projects a second ice puck B across the ice. Referred to the same origin, the position vector of B at time t is

$$\mathbf{r}_B(t) = \tfrac{1}{15}(45 + 75t - t^3)\mathbf{i} + 4\mathbf{j},$$

where $\mathbf{j}$ is a unit vector perpendicular to $\mathbf{i}$.

(i) Find the distance of puck B from the origin when it comes to rest. **4**

(ii) Determine the position of puck A relative to puck B and find the minimum distance between the ice pucks. **5**

[END OF SECTION E]

[END OF QUESTION PAPER]