Homework 11

1) A particle performs simple harmonic motion in a straight line between points A and B with period 0·6 seconds. Initially it was projected from C, the midpoint of AB, with speed $\frac{\pi}{5}$ m s^{-1} towards B.

Calculate

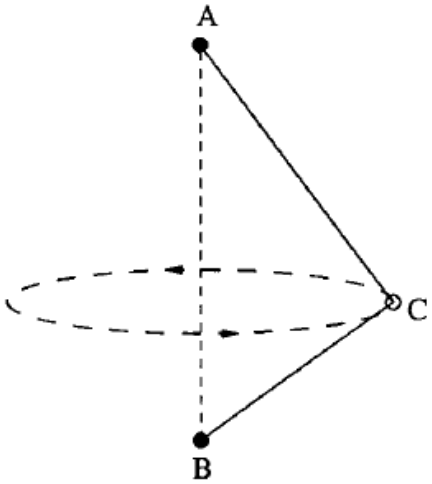
(a) the length of AB, and **3**

(b) the time taken by the particle to move directly from C to D, the midpoint of CB. **2**

2) The maximum speed of a particle executing simple harmonic motion is 13 centimetres per second. When the particle has moved 15 centimetres from its centre of oscillation, its speed is 12 centimetres per second.

Calculate the period and the amplitude of the motion. **5**

3) An inextensible string of length 70 centimetres is attached to two fixed points A and B, where B is 50 centimetres vertically below A. A smooth ring, C, of mass 100 grams, is free to slide on the string. The ring is pulled aside until the string is taut. The ring is then projected so that it moves in a horizontal circle about the line AB, with constant angular speed, as indicated in the diagram.



Given that AC is 40 centimetres, calculate the angular speed of the ring. **7**

4) Differentiate the following functions, simplifying where possible:

(*a*) $f(x) = \dfrac{1+\sin x}{1+2\sin x}$, $0 \le x \le \pi$; 3

(*b*) $g(x) = \ln(1+e^{2x})$. 2

5) In a fairground game, a small target T executes simple harmonic motion about a point O with extreme points A and B. When the target is 1 metre from O, its speed is $\dfrac{\pi}{\sqrt{3}}$ m s^{-1} and when it is $\sqrt{3}$ metres from O its speed is $\dfrac{\pi}{3}$ m s^{-1}.

(*a*) Show that the amplitude of the motion is 2 metres **4**

(b) Calculate the period of the oscillation. **1**