Homework 12

1) A particle executes simple harmonic motion about a point O. The magnitude of the maximum acceleration is 1 $m\,s^{-2}$ and the maximum speed is 4 $m\,s^{-1}$.

Calculate the period of the motion. 4

2) A conical pendulum consists of a bobbin of mass m kilograms attached to one end, B, of a light elastic string AB of natural length l metres and modulus of elasticity $8mg$ newtons. The other end, A, of the string is held fixed. The bobbin moves in a horizontal circle with centre vertically below A, such that the angle between the string AB and the vertical is 45°.

(a) Determine, in terms of l, the extension of the string beyond its natural length. 3

(b) Show that the angular speed, ω radians per second, of the bobbin is given by

$$\omega^2 = \frac{8g}{(1+4\sqrt{2})l}.$$ 3

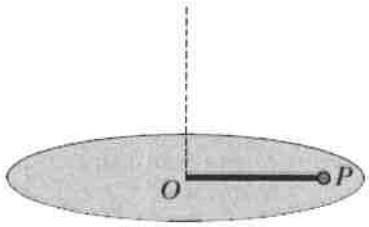
3) Express $\dfrac{x^2+3}{x(1+x^2)}$ in partial fractions. 3

Hence obtain $\displaystyle\int_{1/2}^{1} \frac{x^2+3}{x(1+x^2)}\,dx$. 3

For the function defined by $y = x^2 \ln x$, $x > 0$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. 4

Hence show that $x\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = kx$, stating the value of the constant k. 2

4) A rough disc rotates in a horizontal plane with a constant angular velocity ω about a fixed vertical axis through the centre O. A particle of mass m kilograms lies at a point P on the disc and is attached to the axis by a light elastic string OP of natural length a metres and modulus of elasticity $2mg$.



The particle is at a distance of $\frac{5a}{4}$ from the axis and the coefficient of friction between P and the disc is $\frac{3}{20}$.

Find the range of values for ω such that the particle remains stationary on the disc. 5

5) A toy car of mass 250 grams is stationary on a smooth horizontal surface. One end of a light spring is attached to the car, the other end is fixed to the surface. The natural length of the spring is 1 metre and the modulus of elasticity is 4 newtons.

The car is pulled along the surface, extending the spring by 20 centimetres, and then released.

(*a*) Show that the displacement, x metres, of the car from its equilibrium position satisfies an equation of the form

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

where the value of the constant ω should be stated. 3

(*b*) Calculate the maximum speed of the car. 2