# Homework 13

1) Newton's Inverse Law $\quad a = \frac{k}{r^2}$

at surface of planet $r_p = 8600km$ $\quad a = 11.2ms^{-2}$
$= 8.6 \times 10^6$ metres

$\Rightarrow \quad 11.2 = \frac{k}{(8.6 \times 10^6)^2}$

$k = 8.28 \times 10^{14}$ ✓

For satellite radius of orbit $r_s = 9.2 \times 10^6$ metres

$\frac{k}{r_s^2} = w^2 r_s$ ✓ $\qquad w = 2\pi / T$

$w^2 = \frac{k}{r_s^3}$ ✓ $\qquad T = 2\pi / w$ ✓

$w = 1.03 \times 10^{-3}$ $\qquad T = 6093$ secs.

(T = 1 hr 41 mins)

2)


$T = mg$

$\frac{\lambda e}{l} = mg$ ✓

$\frac{\lambda \times 0.14}{0.8} = 50g$

$\lambda = 2800N$ ✓

b)


$\downarrow m\ddot{y}$

$$m\ddot{y} = mg - T$$

$$m\ddot{y} = mg - \frac{\lambda(e+y)}{l}$$

$$50\ddot{y} = 50g - \frac{2800(0.14+y)}{0.8}$$

$$50\ddot{y} = -3500y$$

$$\underline{\ddot{y} = -70y}$$

c) $\omega = \sqrt{70}$

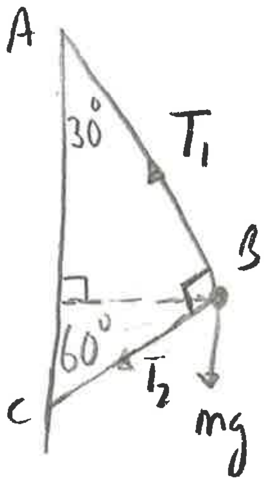
$$v^2 = \omega^2(a^2 - x^2)$$

$$v^2 = 70(0.2^2 - 0.05^2)$$

$$\underline{v = 1.62\,ms^{-1}}$$

[When particle has travelled 15cm it is 5cm from its equilibrium position, hence $x = 5cm = 0.05$ metres]

3)




$$T_1 = \frac{\lambda x}{l}$$

(3)


$$\tan 60 = \frac{x}{L}$$

$$\sqrt{3} = \frac{x}{L}$$

$$x = \sqrt{3}L$$

a) $T_1 = \frac{\lambda e}{l}$  $e = \sqrt{3}L - L$

$$T_1 = \frac{2mg(\sqrt{3}L - L)}{L}$$

$$T_1 = \frac{2mgL(\sqrt{3}-1)}{L}$$

$$\underline{T_1 = 2(\sqrt{3}-1)mg}$$

b) resolve forces vertically

$$T_1 \cos 30 = mg + T_2 \cos 60$$

$$mg\,2(\sqrt{3}-1) \times \frac{\sqrt{3}}{2} = mg + \frac{1}{2}T_2$$

$$\sqrt{3}(\sqrt{3}-1)mg = mg + \frac{1}{2}T_2$$

$$3mg - \sqrt{3}mg - mg = \frac{1}{2}T_2$$

$$\frac{1}{2}T_2 = 2mg - \sqrt{3}mg$$

$$T_2 = (4 - 2\sqrt{3})mg \quad (T_2 = 0.536mg)$$

④

c) $\Sigma F = ma$


$T_1 \sin 30 + T_2 \sin 60 = m\omega^2 r$ ✓

$\sin 60° = \frac{r}{1}$

$r = \sin 60°$

$r = \frac{\sqrt{3}}{2}$ ✓

$2(\sqrt{3}-1)\cancel{m}g \times \frac{1}{2} + (4-2\sqrt{3})\cancel{m}g \times \frac{\sqrt{3}}{2} = \cancel{m}\omega^2 \times \frac{\sqrt{3}}{2}$ ✓

$(\sqrt{3}-1)g + (4-2\sqrt{3})g \times \frac{\sqrt{3}}{2} = \omega^2 \times \frac{\sqrt{3}}{2}$

$2(\sqrt{3}-1)g + \sqrt{3}(4-2\sqrt{3})g = \sqrt{3}\omega^2$

$(2\sqrt{3} - 2 + 4\sqrt{3} - 6)g = \sqrt{3}\omega^2$

$\frac{(6\sqrt{3}-8)g}{\sqrt{3}} = \omega^2$

$\omega = 3.68 \text{ rad s}^{-1}$ ✓

④ $\frac{dy}{dx} = \frac{4x}{4x^2+1}$

$y = \int \frac{4x}{4x^2+1}\,dx$ ✓

$y = \frac{1}{2}\int \frac{8x}{4x^2+1}\,dx$

$y = \frac{1}{2}\ln(4x^2+1) + c$ ✓

at $x = 0$ $y = 0$ $\Rightarrow$ $c = 0$

(5)

$$y = \frac{1}{2}\ln(4x^2+1)$$

$$2y = \ln(4x^2+1)$$

$$e^{2y} = 4x^2+1$$

$$4x^2 = e^{2y} - 1$$

$$x^2 = \frac{e^{2y}-1}{4}$$

$$x = \sqrt{\frac{e^{2y}-1}{4}}$$

$$x = \frac{\sqrt{e^{2y}-1}}{2}$$