Homework 14

1) In vehicle safety trials, a car of mass 1200 kilograms and a van of mass 2000 kilograms were made to crash into each other on a skid-pan. The two vehicles locked together and moved on as one combined mass.

Before impact, taking the x-axis as the direction of the car, the velocity of the car was $18i$ m s^{-1} and the velocity of the van was $(5i + 12j)$ m s^{-1}, where i, j are the unit vectors in the directions of the rectangular axes Ox and Oy respectively.

Given that friction can be ignored, calculate the speed and direction of the combined mass immediately after the crash.

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2) A snooker ball of mass 150 grams is struck by a snooker cue. The magnitude of the contact force, $F(t)$ newtons, between the cue and the ball can be modelled by the expression,

$$F(t) = 12 \sin (20\pi t), \qquad 0 \leqslant t \leqslant 0.05,$$

where t seconds is the time from the start of the impact.

Given that the ball is initially at rest and assuming no spin is imparted to the ball, calculate the speed of the ball 0·05 seconds after the start of the impact.

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3) A satellite orbiting the Earth is required to have a period of two hours. Assume that the orbit is circular and that the only force acting on the satellite is that due to Newton's inverse square law of gravitation. Given that the radius of the Earth is 6380 kilometres, calculate the required altitude of the satellite above the surface of the Earth.

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4) Two particles, P and Q, of masses $2\,\text{kg}$ and $m\,\text{kg}$ respectively, initially lie at rest in a straight line on a smooth horizontal surface. Particle P is acted on by a constant force of $3\,\text{N}$ for 4 seconds, causing it to accelerate towards Q. When P collides with Q the particles coalesce and begin to move with speed $3{\cdot}75\,\text{m s}^{-1}$.

Find the value of m, the mass of particle Q. **4**

5) Express $\dfrac{8}{x(x+2)(x+4)}$ in partial fractions. **4**

Calculate the area under the curve

$$y = \frac{8}{x^3 + 6x^2 + 8x}$$

between $x = 1$ and $x = 2$. Express your answer in the form $\ln\dfrac{a}{b}$, where a and b are positive integers. **5**