Homework 14 solutions

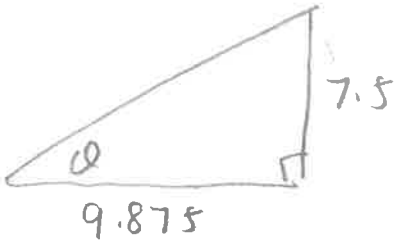
1) momentum before collision = momentum after collision

$1200(18\underline{i}) + 2000(5\underline{i} + 12\underline{j}) = 3200\underline{v}$ ✓

$\underline{v} = 9.875\underline{i} + 7.5\underline{j}$ ✓

$\Rightarrow |\underline{v}| = \sqrt{(9.875)^2 + 7.5^2}$

$= 12.4 ms^{-1}$ ✓



$\tan\theta = \frac{7.5}{9.875}$

$\theta = 37.2°$ ✓

speed is $12.4 ms^{-1}$ at an angle of $37.2°$ to the horizontal

2) $\underline{I} = \int F\,dt$ = change in momentum

$\int_0^{0.05} 12\sin(20\pi)t\,dt = mv - mu$ ✓

$\left[\frac{-12}{20\pi}\cos(20\pi)t\right]_0^{0.05} = 0.15v - 0$ ✓

$0.191 - [-0.191] = 0.15v$

$v = 2.55 ms^{-1}$ ✓

3) $a = \frac{k}{r^2}$

r_E = radius of Earth
$= 6.38 \times 10^6 m$

$9.8 = \frac{k}{(6.38 \times 10^6)^2}$ ✓

$k = 3.99 \times 10^{14}$ ✓

$\frac{k}{r_s^2} = \omega^2 r_s$ ✓

r_s = radius of orbit of satellite

$T = 2 hours$
$= 7200 secs$

$\frac{3.99 \times 10^{14}}{r_s^3} = \omega^2$

$\frac{3.99 \times 10^{14}}{r_s^3} = \left(\frac{2\pi}{T}\right)^2$ ✓

$r_s^3 = 5.24 \times 10^{20}$

$r_s = 8061 km$ ✓

$8061 - 6380 = 1680 km$ above surface ✓

Particle P

4) $I = f \times t$

$= 3 \times 4$

$= 12\,Ns$ ✓

$I = mv - mu$

$12 = 2v$

$v = 6\,ms^{-1}$ ✓

so now use conservation of momentum

$2 \times 6 + 0 = (m+2) \times 3.75$ ✓

$12 = 3.75m + 7.5$

$m = 1.2\,kg$ ✓

5) $\frac{8}{x(x+2)(x+4)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+4}$ ✓

$8 = A(x+2)(x+4) + Bx(x+4) + Cx(x+2)$

$x = 0 \Rightarrow \quad 8 = 8A$

$A = 1$ ✓

$x = -2 \Rightarrow \quad 8 = B \times -2 \times 2$

$B = -2$ ✓

$x = -4 \Rightarrow \quad 8 = C \times -4 \times -2$

$C = 1$

$\Rightarrow \frac{8}{x(x+2)(x+4)} = \frac{1}{x} - \frac{2}{x+2} + \frac{1}{x+4}$ ✓

$\int_1^2 \frac{8}{x^3+6x^2+8x} = \int_1^2 \frac{1}{x} - \frac{2}{x+2} + \frac{1}{x+4}\,dx$ ✓

$= \left[\ln x - 2\ln(x+2) + \ln(x+4)\right]_1^2 = \left[\ln x - \ln(x+2)^2 + \ln(x+4)\right]_1^2$ ✓

$= \left[\ln\left[\frac{x(x+4)}{(x+2)^2}\right]\right]_1^2 = \ln\frac{12}{16} - \ln\frac{5}{9} = \ln\left[\frac{3}{4} \times \frac{9}{5}\right]$

$= \ln\frac{27}{20}$ ✓