Homework 15

1) A body of mass 2kg starts from rest at the origin. It moves in a horizontal straight line with acceleration $(6 - 2t)\mathbf{i}$ m s^{-2}, where t is the time in seconds from the start of the motion and $\mathbf{i}$ is the unit vector in the direction of motion. Find the work done by the total force acting on the body during the interval in which the body moves from rest to its maximum speed. **6**

2) The point of application of the force $\mathbf{F} = 4\mathbf{i} + 11\mathbf{j}$ moves in a straight line from the point $8\mathbf{i} + 5\mathbf{j}$ to the point $32\mathbf{i} + 14\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors. Given that the force is measured in newtons and the displacement in metres, calculate the work done. **3**

3) A particle of mass 2 kg is accelerated horizontally from rest at a point O by a force $8t\mathbf{i}$, whose magnitude is measured in newtons and where $\mathbf{i}$ is the unit vector in the direction of motion and t seconds is the time from the start of the motion.

(*a*) Find the velocity, $\mathbf{v}$, of the particle as a function of time t. **2**

(*b*) Calculate the work done on the particle in the first second of the motion. **3**

4) A railway truck of mass $3m$ kilograms travelling at u m s^{-1} along a straight horizontal track, collides and couples with a stationary truck of mass m kilograms. Due to the action of a constant resistive force of magnitude R newtons, the two trucks come to rest T seconds after the collision.

(*a*) Determine an expression for R in terms of m, u and T. **4**

(*b*) Find an expression, in terms of m and u, for the work done by R in bringing the trucks to rest. **3**

5) A particle with mass 0·25 kg moves along a straight line. Its velocity $\mathbf{v}$ is given by

$$\mathbf{v} = 8\,(1 - e^{-2t})\mathbf{i}$$

where $\mathbf{i}$ is a unit vector in the Ox direction of a rectangular coordinate system with origin O. The time t is measured in seconds and the speed is measured in metres per second.

Obtain an expression for the force acting on the particle at time t. **2**

Show that the work done by this force during the time interval $0 \leq t \leq 1$ is given by

$$32\int_0^1 (e^{-2t} - e^{-4t})\,dt \text{ joules},$$

and hence calculate this value. **4**