

# Homework 15

①

1)  $a = 6 - 2t$

max speed when  $a = 0 \Rightarrow t = 3 \text{ sec}$  ✓

$$a = 6 - 2t$$

$$v = 6t - t^2 + c \text{ at } t = 0 \text{ } v = 0 \Rightarrow c = 0$$

$$v = 6t - t^2 \text{ ✓}$$

$$F = ma$$

$$= 2(6 - 2t)$$

$$F = 12 - 4t \text{ ✓}$$

$$s = 3t^2 - t^3$$

so

$$W = \int_0^3 F \cdot v \, dt$$

$$= \int_0^3 (12 - 4t)(6t - t^2) \, dt \text{ ✓}$$

$$= \int_0^3 (72t - 12t^2 - 24t^2 + 4t^3) \, dt$$

$$= \int_0^3 (72t - 36t^2 + 4t^3) \, dt$$

$$= \left[ 36t^2 - 12t^3 + t^4 \right]_0^3 \text{ ✓}$$

$$= 81 \text{ J} \text{ ✓}$$

at  $t = 3$

$$v = 6t - t^2$$

$$= 18 - 9$$

$$= 9 \text{ ms}^{-1} \text{ ✓}$$

or

$$W = \Delta E$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \text{ ✓}$$

$$= \frac{1}{2} \times 2 \times 9^2 - 0 = 81 \text{ J} \text{ ✓}$$

$$2) \quad F = 4\hat{i} + 11\hat{j}$$

$$\begin{pmatrix} 8 \\ 5 \end{pmatrix} \rightarrow \begin{pmatrix} 32 \\ 14 \end{pmatrix} = \begin{pmatrix} 24 \\ 9 \end{pmatrix}$$

so displacement  $s = 24\hat{i} + 9\hat{j}$

$$W = \underline{F \cdot s}$$

$$= \begin{pmatrix} 4 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 24 \\ 9 \end{pmatrix}$$

$$= 96 + 99$$

$$= \underline{195 \text{ J}}$$

3)



$$ma = 8t\hat{i}$$

$$2a = 8t\hat{i}$$

$$a = 4t\hat{i}$$

$$v = 2t^2\hat{i} + c \quad \text{at } t=0 \quad v=0$$

$$\Rightarrow c=0$$

$$\underline{v = 2t^2\hat{i}}$$

b) at  $t=0 \quad v=0$

at  $t=1 \quad v=2$

work done =  $\Delta E$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2} \times 2 \times 2^2 - 0$$

$$= \underline{4 \text{ J}}$$

or  $W = \int F \cdot v \, dt$

$$= \int_0^1 8t \cdot 2t^2 \, dt$$

$$= \int_0^1 16t^3 \, dt = \left[ \frac{16t^4}{4} \right]_0^1$$

$$= \underline{4 \text{ J}}$$

4) a) momentum before = momentum after  
 $3m \times u + m \times 0 = 4mV$

$$3mu = 4mV$$

$$V = \frac{3u}{4}$$



$$4ma = -R$$

$$a = -\frac{R}{4m}$$

$$v = u + at$$

$$0 = \frac{3u}{4} - \frac{RT}{4m}$$

$v = 0$  when  $t = T$

and  $a = -\frac{R}{4m}$

$$\frac{3u}{4} = \frac{RT}{4m}$$

$$R = \frac{3mU}{T}$$

$$a = -\frac{R}{4m}$$

$$a = -\frac{3mU}{4mT}$$

$$a = -\frac{3U}{4T}$$

$$b) S = ut + \frac{1}{2}at^2$$

$$S = \frac{3uT}{4} + \frac{1}{2} \left( \frac{-3U}{4T} \right) T^2$$

$$S = \frac{3UT}{4} - \frac{3UT}{8}$$

$$S = \frac{3UT}{8}$$

$$W = F \times d$$

$$= \frac{3mU}{T} \times \frac{3UT}{8}$$

$$W = \frac{9mU^2}{8}$$

5)  $v = 8(1 - e^{-2t})$

$v = 8 - 8e^{-2t}$

$a = 16e^{-2t}$

$F = ma$   
 $F = 0.25 \times 16e^{-2t}$   
 $F = 4e^{-2t}$  ✓

$v = 8 - 8e^{-2t}$  ✓

$W = \int F \cdot v \, dt$   
 $= \int_0^1 4e^{-2t}(8 - 8e^{-2t}) \, dt$  ✓  
 $= \int_0^1 32e^{-2t} - 32e^{-4t} \, dt$   
 $= 32 \int_0^1 (e^{-2t} - e^{-4t}) \, dt$  ✓

$W = 32 \left[ -\frac{1}{2}e^{-2t} + \frac{1}{4}e^{-4t} \right]_0^1$  ✓

$W = 5.98 \text{ J}$  ✓