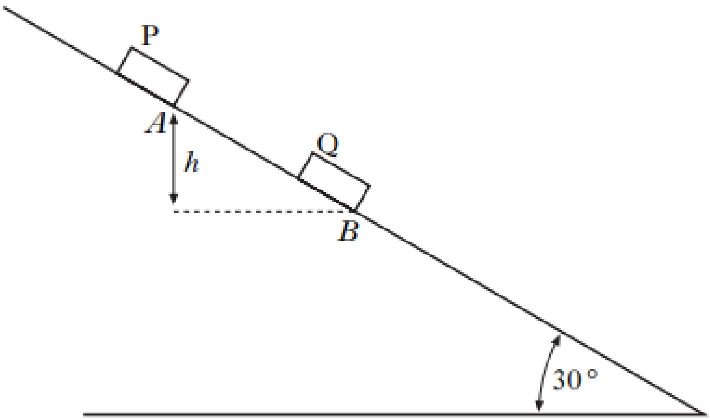
Homework 16

1) . Ben is cycling up a straight road which is inclined at an angle θ to the horizontal where $\sin\theta = \frac{1}{20}$. The combined mass of Ben and the cycle is 100 kg. The resistance to the motion from non-gravitational forces is a force of magnitude kv^2 newtons, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the cycle and k is a constant.

When Ben is cycling up the road at $2\,\mathrm{m\,s^{-1}}$, his acceleration is $0{\cdot}05\,\mathrm{m\,s^{-2}}$ and the rate at which he is working is 120 W.

Calculate the value of the constant k. **6**

2) The diagram shows a ramp, inclined at 30° to the horizontal, which has a smooth section above B and a rough section below B. Identical blocks, P and Q, each has weight W newtons. Block Q is stationary at B, held by friction, and block P is held at rest at A. Block P is a vertical height of h metres above block Q (where the dimensions of the blocks should be ignored).



When block P is released, it slides down the ramp colliding and coupling with block Q. The combined blocks then move down the rough section of the ramp, coming to rest at a vertical height $\frac{1}{2}h$ metres below B.

(i) Find, in terms of g and h, the speed of the combined block immediately after the collision. **3**

(ii) Using the work/energy principle, show that the constant frictional force acting on the combined block whilst it is moving has magnitude $\frac{3}{2}W$ newtons. **4**

3)

A mass m kilograms is attached to one end, A, of a light inextensible string of length L metres, the other end of which is fixed at a point O. Initially the mass hangs vertically below O with the string taut. The mass is then given a horizontal speed of $\sqrt{\frac{7}{2}gL}$ ms^{-1}, causing it to start to travel in a vertical circle of centre O. Subsequently, the string OA makes an angle θ with the downward vertical through O.

(*a*) When $\theta = 45°$, find expressions for:

(i) the speed of the mass in terms of L and g; **4**

(ii) the magnitude of the tension in the string, in terms of m and g. **3**

(*b*) Determine the value of θ at which the string first becomes slack. **4**

4)

. Find the exact value of $\int_2^7 \frac{x}{\sqrt{x+2}}dx$ using the substitution $u = x + 2$. **5**