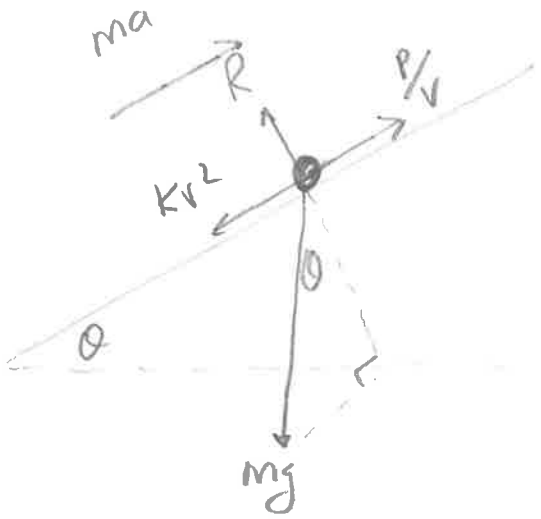
Homework 16

1)

$P = F \times v$

$F = P/v$



$\sin\theta = \frac{1}{20}$

$ma = \frac{P}{v} - mg\sin\theta - kv^2$ ✓

$100 \times 0.5 = \frac{120}{2} - 100g \times \frac{1}{20} - k \times 2^2$ ✓

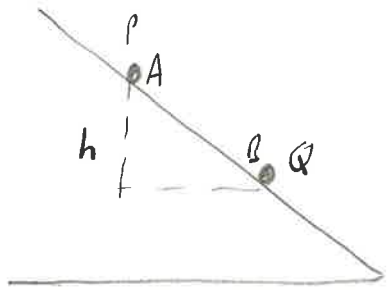
$5 = 60 - 5g - 4k$ ✓

$5 = 11 - 4k$

$4k = 6$

$k = 1.5$ ✓

2) i)



immediately before collision

At A

$E_p = mgh$

At B

$E_K = \frac{1}{2}mv^2$

$\Rightarrow \quad \frac{1}{2}mv^2 = mgh$

$v^2 = 2gh$

$v = \sqrt{2gh}$ ✓

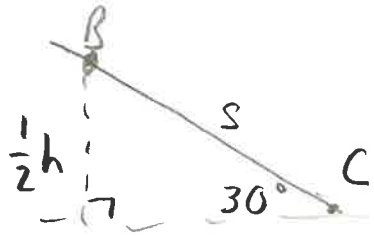
now use conservation of momentum

momentum immediately before collision = momentum immediately after collision

$m \times \sqrt{2gh} + m \times 0 = 2m \times V$ ✓

$V = \frac{\sqrt{2gh}}{2}$ ✓

ii)



work done = ΔE

$F \times S$ = Energy at B − Energy at C

At B

$E_K = \frac{1}{2} \times 2m \times v^2$

$= \frac{1}{2} \times 2m \times \frac{2gh}{4}$

$= \frac{1}{2}mgh$ ✓ $(= \frac{1}{2}Wh)$

$E_p = 2m \times g \times \frac{1}{2}h$

$= mgh$ ✓ $(= Wh)$

At C

Energy = 0

it has stopped

$\sin 30° = \frac{\frac{1}{2}h}{S}$

$h = S$

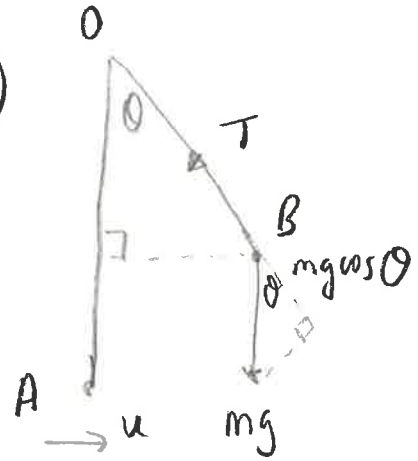
so

$F \times S = \frac{1}{2}mgh + mgh - 0$ ✓

$F \times K = \frac{3}{2}mgk$

$F = \frac{3}{2}mg = \frac{3}{2}W$ ✓

3a)



conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$$ ✓✓

$$u^2 = v^2 + 2gr(1-\cos\theta)$$

$u = \sqrt{\frac{7}{2}gL}$, $\theta = 45°$ $\Rightarrow$

$$\frac{7}{2}gL = v^2 + 2gL(1-\cos 45)$$

$$\frac{7}{2}gL = v^2 + (2-\sqrt{2})gL$$

$$v^2 = \frac{3+2\sqrt{2}}{2}gL$$ ✓ [$v^2 = 2.91gL$]

$$v = \sqrt{\frac{3+2\sqrt{2}}{2}gL}$$

$$v = 1.71\sqrt{gL}$$ ✓

b) $\sum F = ma$

$ma = T - mg\cos\theta$

$\frac{mv^2}{r} = T - mg\cos\theta$ ✓

$T = \frac{mv^2}{r} + mg\cos\theta$

$T = \frac{m}{k}\left(\frac{3+2\sqrt{2}}{2}gk\right) + mg\cos\theta$ ✓

$T = \frac{3+3\sqrt{2}}{2}mg$

$T = 3.62\,mg$ ✓

c) string goes slack when $T = 0 \Rightarrow \frac{mv^2}{r} = -mg\cos\theta$

$v^2 = -gr\cos\theta$ ✓

$u^2 = v^2 + 2gr(1-\cos\theta)$ ✓

$\Rightarrow \quad u^2 = -gr\cos\theta + 2gr(1-\cos\theta)$

$\frac{7}{2}gk = -gk\cos\theta + 2gk(1-\cos\theta)$

$\frac{7}{2} = 2 - 3\cos\theta$

$3\cos\theta = -\frac{3}{2}$

$\cos\theta = -\frac{1}{2}$ ✓

$\theta = 120^\circ \left(\theta = \frac{2\pi}{3} \text{ radians}\right)$ ✓

4) $\int_2^7 \frac{x}{\sqrt{x+2}}\,dx$

$u = x+2$

$du = dx$

$x = u-2$

$x\Big|_2^7 \rightarrow u\Big|_4^9$ ✓

$= \int_4^9 \frac{u-2}{u^{1/2}}\,du$ ✓

$= \int_4^9 \frac{u}{u^{1/2}} - \frac{2}{u^{1/2}}\,du$

$= \int_4^9 (u^{1/2} - 2u^{-1/2})\,du$ ✓

$= \left[\frac{u^{3/2}}{3/2} - \frac{2u^{1/2}}{1/2}\right]_4^9$ ✓

$= \left[\frac{2u^{3/2}}{3} - 4u^{1/2}\right]_4^9$

$= 6 - \left[-\frac{8}{3}\right]$

$= \frac{26}{3}$ ✓