Homework 17 solutions

1) $V_c = 10\underline{i} - 15\underline{j}$

$r_c = 10t\underline{i} - 15t\underline{j} + c$ at $t=0$ $r = 20\underline{i} + 50\underline{j}$

$\Rightarrow$ $r_c = (10t+20)\underline{i} + (50-15t)\underline{j}$ ✓

$V_L = -5\underline{i} + 30\underline{j}$

$r_L = -5t\underline{i} + 30t\underline{j} + c$ at $t=0$ $r = 90\underline{i} + 10\underline{j}$

$\Rightarrow$ $r_L = (90-5t)\underline{i} + (30t+10)\underline{j}$ ✓

$Cr_L = [(10t+20)\underline{i} + (50-15t)\underline{j}] - [(90-5t)\underline{i} + (30t+10)\underline{j}]$

$Cr_L = (15t-70)\underline{i} + (40-45t)\underline{j}$ ✓

if collide then $Cr_L = 0$ [relative distance between car and lorry = 0]

$15t_1 - 70 = 0$ $\quad 40 - 45t_2 = 0$

$t_1 = 70/15$ $\quad t_2 = 40/45$

$t_1 \neq t_2$ so don't collide ✓

b) $(Cr_L)^2 = (15t-70)^2 + (40-45t)^2$ ✓

$= 225t^2 - 2100t + 4900 + 1600 - 3600t + 2025t^2$

$= 2250t^2 - 5700t + 6500$

$\frac{d(Cr_L)^2}{dt} = 4500t - 5700$ ✓

at min $\frac{d(Cr_L)^2}{dt} = 0$

$4500t - 5700 = 0$

$t = 1.27$ ✓

$(Cr_L)^2 = 2890$

$\Rightarrow (Cr_L) = \sqrt{2890} = 53.8\,km$ ✓

2a) vertical motion

$u = 8$
$v = 0$
$a = -9.8$
$s = ?$

$v^2 = u^2 + 2as$ ✓

$0 = 8^2 - 2 \times 9.8s$ ✓

$s = 3.27m$

b) vertical motion

$u = 8$
$v = 0$
$a = -9.8$
$t = ?$

$v = u + at$

$0 = 8 - 9.8t$

$t = 0.816$ secs ✓

total time $= 2 \times 0.816$ ✓
$= 1.63$ secs

distance = horizontal speed × time
$= 10 \times 1.63$
$= 16.3m$ ✓

3) $x \sin y + y \cos x = 2$

$1 \cdot \sin y + x \cdot \cos y \frac{dy}{dx} + \frac{dy}{dx} \cdot \cos x + y \cdot -\sin x = 0$

$\sin y + x \cos y \frac{dy}{dx} + \cos x \frac{dy}{dx} - y \sin x = 0$

$(x \cos y + \cos x) \frac{dy}{dx} = y \sin x - \sin y$

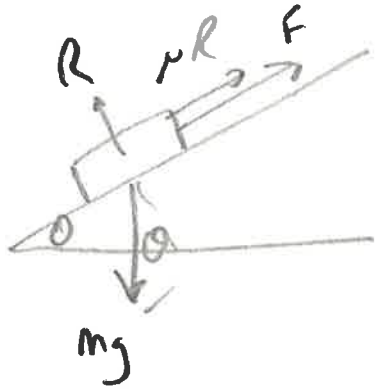
$x \sin y + y \cos x = 2$

so at $x = 0 \Rightarrow y = 2$

$(x \cos y + \cos x) \frac{dy}{dx} = y \sin x - \sin y$

at $x = 0$, $y = 2$ $\Rightarrow$ $1 \cdot \frac{dy}{dx} = -\sin 2$

4a)



on the point of sliding down so μR is acting up

$F + \mu R = mg \sin\theta \quad R = mg \cos\theta$

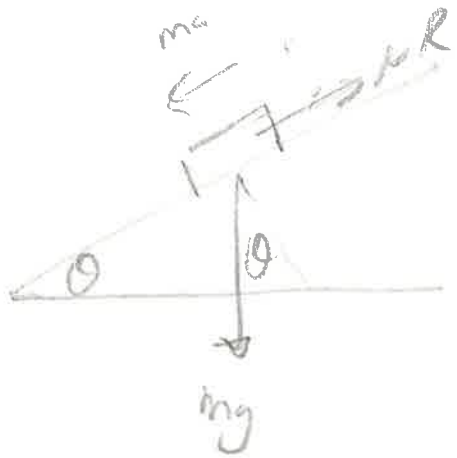
$F + \mu mg \cos\theta = mg \sin\theta$

$\mu = \frac{mg \sin\theta - F}{mg \cos\theta} \quad (F = 4N)$

$\mu = 0.277$

(4)

b)



$ma = mg\sin\theta - \mu R$ ✓ $\qquad R = mg\cos\theta$

$a = g\sin\theta - \mu g\cos\theta$ ✓

$a = 0.8\,ms^{-2}$ ✓

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2\times 0.8 \times 2$

$v = 1.79\,ms^{-1}$ ✓