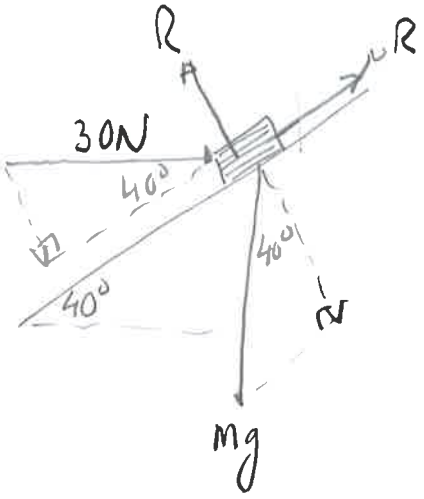
Homework 19 solutions

1a)



in equilibrium

resolve in direction of slope

$\mu R + 30\cos 40 = mg\sin 40$ ✓

resolve perpendicular to slope

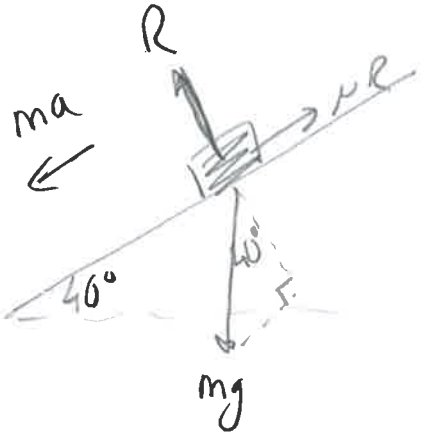
$R = 30\sin 40° + mg\cos 40°$ ✓

$\Rightarrow \quad \mu(30\sin 40° + mg\cos 40°) + 30\cos 40 = mg\sin 40$ ✓

$$\mu = \frac{mg\sin 40 - 30\cos 40}{30\sin 40 + mg\cos 40}$$

$\mu = 0.54$ ✓

b)



$\sum F = ma$

$ma = mg\sin 40 - \mu R \qquad R = mg\cos 40$ ✓

$ma = mg\sin 40 - \mu mg\cos 40$ ✓

$a = 2.2\ ms^{-2}$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 2.2 \times 3$

$v = 3.66\ ms^{-1}$ ✓

2a) $x = 0.3m$
$v = 5ms^{-1}$

$T = 1.5$

$\omega = 2\pi/T$

$\omega = 2\pi/1.5$

$\omega = 4\pi/3$

$v^2 = \omega^2(a^2 - x^2)$

$5^2 = \left(\frac{4\pi}{3}\right)^2(a^2 - 0.3^2)$ ✓

$a^2 = 1.51$

$a = 1.23m$ ✓

b) $V_{max} = \omega a$

$= \frac{4\pi}{3} \times 1.23$

$= 5.2 ms^{-1}$ ✓

3) $\int e^{4x} \sin 2x \, dx$ (u v'; $uv - \int u'v$)

$= e^{4x} \cdot -\frac{1}{2}\cos 2x - \int 4e^{4x} \cdot -\frac{1}{2}\cos 2x \, dx$

$= -\frac{1}{2} e^{4x} \cos 2x + 2\int e^{4x} \cos 2x \, dx$

$\int e^{4x} \cos 2x \, dx$ (u v'; $uv - \int u'v$)

$= e^{4x} \cdot \frac{1}{2}\sin 2x - \int 4e^{4x} \cdot \frac{1}{2}\sin 2x \, dx$

$= \frac{1}{2} e^{4x} \sin 2x - 2\int e^{4x} \sin 2x \, dx$

$\int e^{4x} \sin 2x \, dx = -\frac{1}{2} e^{4x} \cos 2x + 2\left[\frac{1}{2} e^{4x} \sin 2x - 2\int e^{4x} \sin 2x \, dx\right]$

$\int e^{4x} \sin 2x \, dx = -\frac{1}{2} e^{4x} \cos 2x + e^{4x} \sin 2x - 4\int e^{4x} \sin 2x \, dx$

$5\int e^{4x} \sin 2x = -\frac{1}{2} e^{4x} \cos 2x + e^{4x} \sin 2x$

$\int_0^{\pi/4} e^{4x} \sin 2x = \frac{1}{5}\left[-\frac{1}{2} e^{4x} \cos 2x + e^{4x} \sin 2x\right]_0^{\pi/4}$

$= \frac{1}{5}\left[0 + e^{\pi} - \left(-\frac{1}{2} + 0\right)\right]$

$= \frac{1}{5}\left[e^{\pi} + \frac{1}{2}\right]$

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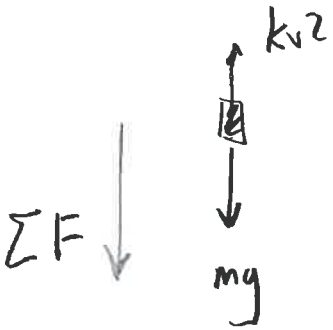
4a)



$\sum F = ma$

$ma = mg - kv^2$ ✓

at max speed $a = 0 \Rightarrow kv^2 = mg$ ✓

$K = 0.25$

b)

$ma = mg - \frac{1}{4}v^2$

$5a = 5g - \frac{1}{4}v^2$ $\qquad a = v\frac{dv}{dx}$

$5v\frac{dv}{dx} = 5g - \frac{1}{4}v^2$ ✓

$20v\frac{dv}{dx} = 20g - v^2$

$\int \frac{20v}{20g - v^2}\,dv = \int dx$ ✓

$$-10\int_0^V \frac{-2v}{20g-v^2}\,dv = \int_0^x dx$$

$$\left[10\ln|20g-v^2|\right]_0^V = [x]_0^x \quad \checkmark$$

$$-10\ln|20g-v^2| - [-10\ln 20g] = x - 0$$

$$10\ln\frac{20g}{20g-v^2} = x$$

$$\ln\frac{20g}{20g-v^2} = 0.1x$$

$$\frac{20g}{20g-v^2} = e^{0.1x} \quad \checkmark$$

$$\frac{20g}{e^{0.1x}} = 20g-v^2$$

$$20ge^{-0.1x} = 20g-v^2$$

$$v^2 = 20g - 20ge^{-0.1x}$$

$$\underline{v^2 = 20g\left(1-e^{-0.1x}\right)} \quad \checkmark$$