Homework 21

1) Use the substitution $u = x + 1$ to obtain $\int \frac{x^2+2}{(x+1)^2}dx$. **5**

2) A rubber ball of mass m kg falls vertically into a tank of water. When the ball is x metres below the surface of the water and moving downwards with speed v m s^{-1}, the water exerts a resistive force of magnitude $2mv^2$ newtons and an upward buoyancy force of magnitude three times the weight of the ball.

(a) Show that the downward motion of the ball can be modelled by the differential equation

$$v\frac{dv}{dx} = -2(v^2 + g).$$ **2**

(b) The ball enters the water with speed U m s^{-1}. By solving the equation in (a), show that

$$v^2 + g = (U^2 + g)e^{-4x}.$$ **5**

(c) In the case when $U = 4{\cdot}9$, calculate, to the nearest centimetre, the greatest depth below the surface of the water reached by the ball. **3**

3) The motion of a spring can be modelled by the differential equation,

$$9\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 4x = 0$$

When $t = 0,\ x = 6$ **and** $\frac{dx}{dt} = 2.$

Determine the particular solution of this differential equation. **6**

4) An object of mass $0 \cdot 2\text{kg}$ is attached to the end of a piece of string of length 15cm.

The object hangs vertically and then is given a push with an initial speed of 1ms^{-1}.

Determine the speed of the object at the point when the string goes slack. **7**