Prelim

(1) $v = 3t^2 - 30t + 72$

at rest $v = 0 \Rightarrow 3t^2 - 30t + 72 = 0$ ✓

$$t^2 - 10t + 24 = 0$$
$$(t-4)(t-6) = 0$$
$$t = 4, 6 \checkmark$$

$$s = t^3 - 15t^2 + 72t + C$$

at $t = 0$ $s = 0 \Rightarrow C = 0$

$$s = t^3 - 15t^2 + 72t \checkmark$$

at $t = 4$ $s = 112m$ ✓ at $t = 6$ $s = 108m$

distance $= \underline{\underline{4m}}$ ✓

(2) $V_P = \underline{i} + 3\underline{j}$ $V_Q = 2\underline{i}$

$S_P = t\underline{i} + 3t\underline{j} + C$ $S_Q = 2t\underline{i} + C$

at $t = 0$ $4\underline{i} + 2\underline{j} = C$ at $t = 0$ $2\underline{i} + 8\underline{j} = C$

$\Rightarrow S_P = (t+4)\underline{i} + (3t+2)\underline{j}$ ✓ $\Rightarrow S_Q = (2t+2)\underline{i} + 8\underline{j}$ ✓

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$S_P = (t+4)\underline{i} + (3t+2)\underline{j}$ $\quad$ $S_Q = (2t+2)\underline{i} + 8\underline{j}$

if collide $S_P = S_Q$ at same value of t.

<u>equate i components</u>

$t + 4 = 2t + 2$

$\underline{t = 2}$ ✓

<u>equate j components</u>

$3t + 2 = 8$

$3t = 6$

$\underline{t = 2}$

$\Rightarrow$ <u>collision</u> ✓

at $t = 2$ $\quad S_P = 6\underline{i} + 8\underline{j}$ ✓

at $t = 0$ $\quad S_P = 4\underline{i} + 2\underline{j}$

distance travelled $= \sqrt{2^2 + 6^2} = \underline{6.32m}$ ✓

3 $V_{max} = wa$

$wa = 45$ ✓

$\Rightarrow w^2a^2 = 2025$

$v^2 = w^2(a^2 - x^2)$

$33.5^2 = w^2(a^2 - 20^2)$ ✓

$1122.25 = w^2a^2 - 400w^2$

$1122.25 = 2025 - 400w^2$ ✓

$400w^2 = 902.75$

$w = 1.50 \text{ rads}^{-1}$

$T = {}^{2\pi}/_{w} \Rightarrow T = {}^{2\pi}/_{1.50}$

$\underline{T = 4.18 \text{ secs}}$ ✓

b) $v_{max} = \omega a$

$45 = 1.50 \times a$

$a = 30.0\text{cm}$

4) $x^2y + y^2 = 10$

$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

when $x = 3$

$x^2y + y^2 = 10$

$9y + y^2 = 10$

$y^2 + 9y - 10 = 0$

$(y+10)(y-1) = 0$

$y = 1$ $\cancel{y = -10}$ $y > 0$

$x = 3, y = 1$

$2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$6 + 9\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

$11\frac{dy}{dx} = -6$

$\frac{dy}{dx} = -6/11 \; (= -0.545)$

(4)

5)

$m = 0.5kg$.

conservation of energy

At A

$E_k = \frac{1}{2}mu^2$

At B

$E_K = \frac{1}{2}mv^2$

$E_p = mgr(1-\cos\theta)$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta) \checkmark$$

$$u^2 = v^2 + 2gr(1-\cos\theta)$$

$$v^2 = u^2 - 2gr(1-\cos\theta) \checkmark$$

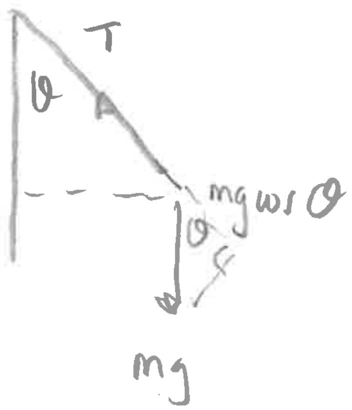
$u = 4$

$\theta = 70°$

$$v^2 = 4^2 - 2\times9.8\times0.6(1-\cos70)$$

$$\underline{v = 2.87ms^{-1}} \checkmark$$

b)



$$\Sigma F = ma$$

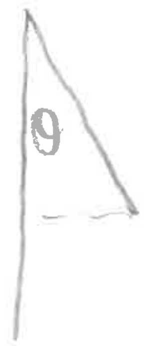
$$\frac{mv^2}{r} = T - mg\cos\theta$$

$$T = \frac{mv^2}{r} + mg\cos\theta \checkmark$$

$$T = \frac{0.5\times2.87^2}{0.6} + 0.5\times9.8\cos70$$

$$\underline{T = 8.56N} \checkmark$$

c)



$$\boxed{v^2 = u^2 - 2gr(1-\cos\theta)}$$

$$T = \frac{mv^2}{r} + mg\cos\theta$$

to go round in a complete circle $T > 0$ when $\theta = 180°$ ✓

at $T = 0$ $\quad \frac{mv^2}{r} + mg\cos\theta = 0$

$$\boxed{v^2 = -gr\cos\theta} \checkmark$$

$\Rightarrow \quad u^2 - 2gr(1-\cos\theta) = -gr\cos\theta$ ✓

$\theta = 180°$ $\quad u^2 - 2gr \times 2 = gr$

$$u^2 = 5gr$$

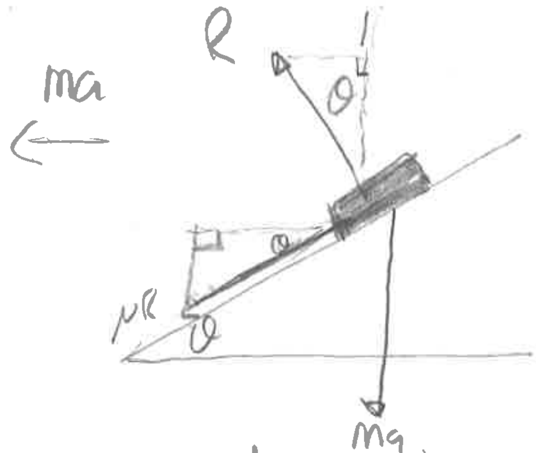
$$u = \sqrt{5gr}$$

so for complete circle $\quad u > \sqrt{5gr}$

$$u > \sqrt{5 \times 9.8 \times 0.6}$$

$$\underline{u > 5.42\,ms^{-1}} \checkmark$$

6



at max speed car is on the point of sliding outwards so friction is acting towards down the slope ✓

$$\sum F = ma$$

$$R\sin\theta + \mu R\cos\theta = \frac{mv^2}{r} \checkmark$$

resolve vertically

$$R\cos\theta = \mu R\sin\theta + mg$$

$$R\cos\theta - \mu R\sin\theta = mg \checkmark$$

$$\frac{\cancel{R}\sin\theta + \mu\cancel{R}\cos\theta = \frac{\cancel{m}v^2}{r}}{\cancel{R}\cos\theta - \mu\cancel{R}\sin\theta = \cancel{m}g} \checkmark$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{gr} \checkmark$$

$$\frac{v^2}{gr} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

$r = 100$
$\mu = 0.3$
$\theta = 8°$

$v = 21.2\,ms^{-1}$ ✓

or $v = 47.8$ mph

so speed limit set at 40mph. ✓

(7) $f(x) = \text{cosec}(x^2)$

$f'(x) = -\text{cosec}(x^2).\cot(x^2).2x$

$f'(x) = -2x\,\text{cosec}(x^2)\cot(x^2)$

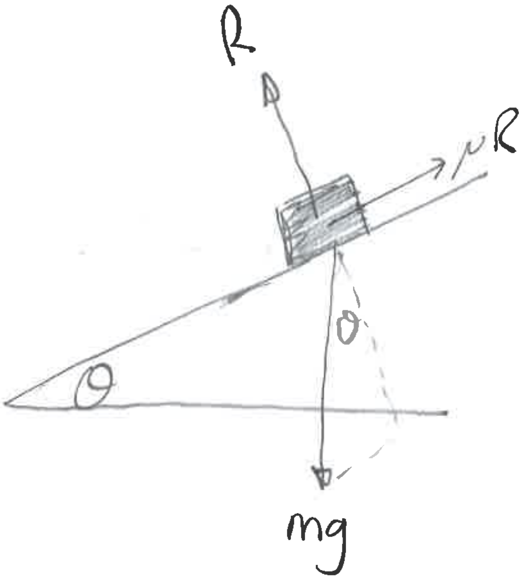
b) $g(x) = \dfrac{(1-x)^2}{\ln x}$

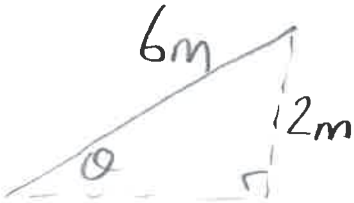
$$g'(x) = \frac{2(1-x).-1\ln x - (1-x)^2.\frac{1}{x}}{(\ln x)^2}$$

$$= \frac{-2\,(1-x)\ln x - \frac{1}{x}(1-x)^2}{(\ln x)^2}$$

$$g'(x) = \frac{-2x\,(1-x)\ln x - (1-x)^2}{x(\ln x)^2}$$ ✓

8)





$\sin\theta = 2/6$

$\sin\theta = 1/3$

$[\theta = 19.5°]$

in equilibrium

$R = mg\cos\theta$

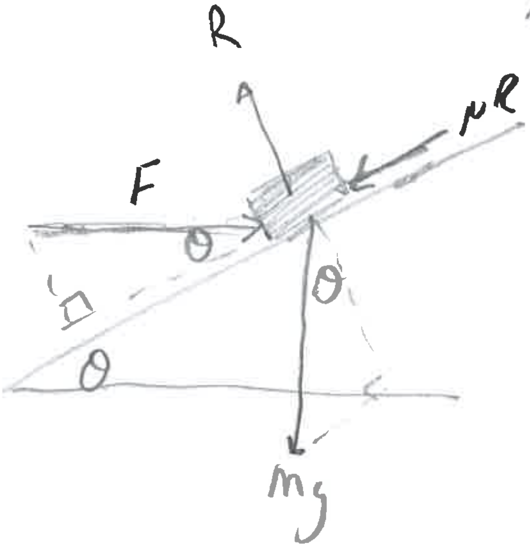
$\mu R = mg\sin\theta$

$\mu \cancel{mg}\cos\theta = \cancel{mg}\sin\theta$

$\mu = \tan\theta$

$\mu = 0.354$

b)



in equilibrium

$F\cos\theta = \mu R + mg\sin\theta$

$R = F\sin\theta + mg\cos\theta$

$$F\cos\theta = \mu R + mg\sin\theta$$

and $R = F\sin\theta + mg\cos\theta$

so $F\cos\theta = \mu(F\sin\theta + mg\cos\theta) + mg\sin\theta$

$$F\cos\theta - \mu F\sin\theta = \mu mg\cos\theta + mg\sin\theta$$

$$F = \frac{\mu mg\cos\theta + mg\sin\theta}{\cos\theta - \mu\sin\theta}$$

$$F = \frac{0.354 \times 40g\cos 19.5 + 40g\sin 19.5}{\cos 19.5 - 0.354\sin 19.5}$$

$$\underline{F = 317N}$$

4) <u>vertical motion</u>

$v^2 = u^2 + 2as$

$0 = (20\sin\theta)^2 - 2g \times 4$

$(20\sin\theta)^2 = 8g$

$400\sin^2\theta = 8g$

$\sin\theta = \sqrt{\frac{8g}{400}}$

$\underline{\theta = 26.3^\circ}$

$s = 4$

$u = 20\sin\theta$

$v = u + at$

$0 = 20\sin 26.3 - gt$

$t = 0.904$ secs

so total time of flight

$= \underline{1.81 \text{ secs}}$

range $= u \times t$

$= 20\cos 26.3 \times 1.81$

$= \underline{32.4\text{m}}$

10)

$$\frac{7x^2 - x + 8}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$$7x^2 - x + 8 = A(x^2+2) + x(Bx+c)$$

$x = 0$ $\qquad 8 = 2A$

$\qquad A = 4$

$x = 1$ $\qquad 14 = 4 \times 3 + B + C$

$\qquad B + C = 2$

$x = 2$ $\qquad 34 = 24 + 4B + 2C$

$\qquad 4B + 2C = 10$

$\qquad 2B + C = 5$

$\qquad B + C = 2$

$\qquad 2B + C = 5$

$\Rightarrow \quad B = 3 \quad \Rightarrow \quad C = -1$

$$\frac{7x^2 - x + 8}{x(x^2+2)} = \frac{4}{x} + \frac{3x-1}{x(x^2+2)}$$

11)

$$a = (2t-1)\underline{i} + 4\underline{j}$$

$$F = ma$$

$$F = 2(2t-1)\underline{i} + 8\underline{j}$$

$$I = \int F\,dt$$

$$I = \int_0^3 [(4t-2)\underline{i} + 8\underline{j}]\,dt$$

$$= \left[(2t^2-2t)\underline{i} + 8t\underline{j}\right]_0^3$$

$$= 12\underline{i} + 24\underline{j} - 0$$

$$I = (12\underline{i} + 24\underline{j})\,Ns$$

b)

$$I = mv - mu$$

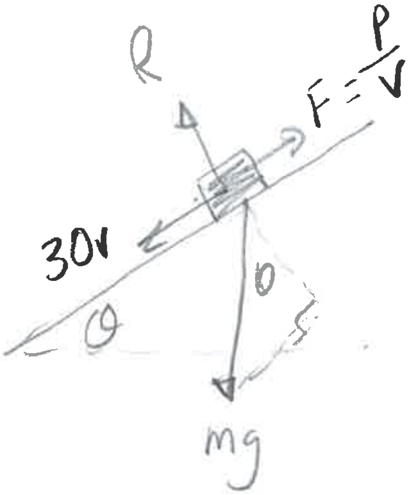
$$12\underline{i} + 24\underline{j} = 2\underline{v} - 2(3\underline{i} - 2\underline{j})$$

$$12\underline{i} + 24\underline{j} = 2\underline{v} - 6\underline{i} + 4\underline{j}$$

$$2\underline{v} = 18\underline{i} + 20\underline{j}$$

$$\underline{v} = (9\underline{i} + 10\underline{j})\,ms^{-1}$$

12)



Constant speed ⇒ forces are balanced.

$$\frac{P}{V} = 30v + mg\sin\theta$$

$$\frac{12800}{v} = 30v + 1000 \times g \times \frac{1}{10}$$

$$\frac{12800}{v} = 30v + 980$$

$$30v^2 + 980v - 12800 = 0$$

$$3v^2 + 98v - 1280 = 0$$

$$v = \frac{-98 \pm \sqrt{24964}}{6}$$

$\underline{v = 10}$ $\cancel{v = -42.7}$ since $v > 0$

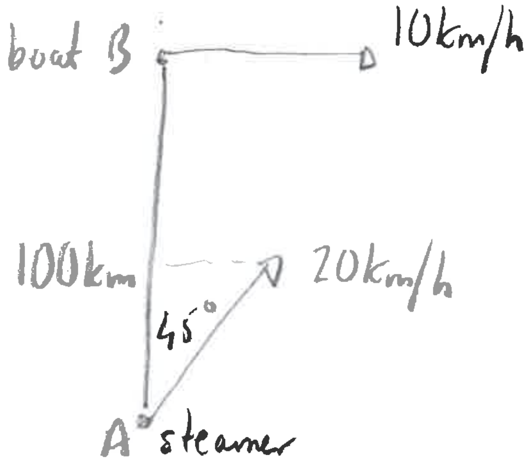
$$\Rightarrow \underline{V = 10\,ms^{-1}}$$

b)

$$ma = \frac{P}{v} - 30v$$

$$a = 0.98\,ms^{-2}$$

13a)



$$V_A = (20\sin 45)\underline{i} + (20\cos 45)\underline{j}$$

$$r_A = 10\sqrt{2}t\underline{i} + 10\sqrt{2}t\underline{j} + C \quad \text{at } t = 0 \text{ position of } A = 0$$

$$\Rightarrow C = 0$$

$$r_A = 10\sqrt{2}t\underline{i} + 10\sqrt{2}t\underline{j}$$

$$V_B = 10\underline{i}$$

$$r_B = 10t\underline{i} + C \quad \text{at } t = 0 \quad 100\underline{j} = C$$

$$r_B = 10t\underline{i} + 100\underline{j}$$

$$A r_B = 10\sqrt{2}t\,\underline{i} + 10\sqrt{2}t\,\underline{j} - [10t\,\underline{i} + 100\,\underline{j}]$$
$$= 4.14t\,\underline{i} + (10\sqrt{2}t - 100)\,\underline{j}$$

b) $$|A r_B|^2 = (4.14t)^2 + (10\sqrt{2}t - 100)^2_j$$
$$= 17.2t^2 + 200t^2 - 2828.4t + 10000$$
$$= 217.2t^2 - 2828.4t + 10000$$

$$\frac{d|A r_B|^2}{dt} = 434.4t - 2828.4$$

at closest $\frac{d|A r_B|^2}{dt} = 0$ $\quad 434.4t - 2828.4 = 0$

$$t = 6.51$$

$$|A r_B| = \sqrt{217.2t^2 - 2828.4t + 10000}$$
$$= \underline{28.1\text{km}}$$

c) $$217.2t^2 - 2828.4t + 10000 = 50^2$$
$$217.2t^2 - 2828.4t + 7500 = 0$$
$$t = \frac{2828.4 \pm \sqrt{1483846}}{434.4}$$

time interval $= \frac{2\sqrt{1483846}}{434.4} = 5.61\text{hrs}$

$= \underline{5\text{hrs } 37\text{mins}}$