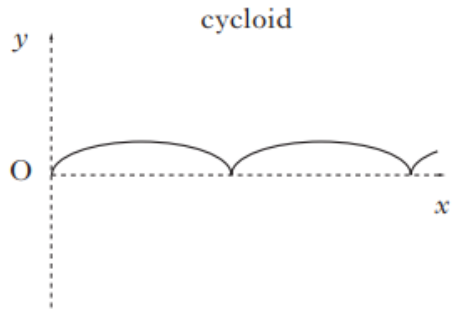
Homework 10

1) The cycloid curve below is defined by the parametric equations

$$x = t - \sin t,\ y = 1 - \cos t.$$

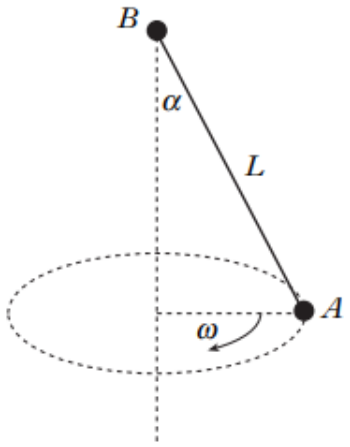


a) Find $\frac{dy}{dx}$ in terms of t. **2**

b) A particle follows the path of the cycloid where t is the time elapsed since the particle's motion commenced.

Calculate the speed of the particle when $t = \frac{\pi}{3}$. **2**

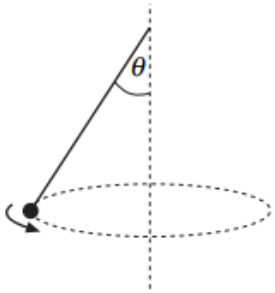
2) A ball of mass m kg is attached to one end A of a light inextensible string of length L metres. The other end of the string is attached to a fixed point B. The ball moves, with string taut, in a horizontal circle with constant angular speed ω radians per second as shown in the diagram. During this motion, the string is inclined at an angle α to the downward vertical through B where $\tan\alpha = \frac{5}{12}$.



(*a*) Find the tension in the string in terms of m and g. **2**

(*b*) Find an expression for ω in terms of g and L. **3**

3)



A particle of mass 2 kg is attached to one end of a light inextensible string of length 2 metres. The other end of the string is held fixed while the mass moves in a horizontal circle about a vertical axis at 5 radians per second.

Calculate the size of angle θ, between the string and the vertical axis. **5**

4)

A bend on a racetrack is circular and is banked at an angle α to the horizontal such that a car can take the bend at speed V with no sideways friction between the wheels and the track. Express V in terms of α, r and g, where r is the radius of the bend and g is the magnitude of the acceleration due to gravity. **3**

A car taking the bend at speed v is on the point of skidding outwards. Show that

$$v^2 = \frac{gr(\tan\alpha + \mu)}{1 - \mu\tan\alpha},$$

where μ is the coefficient of friction between the car tyres and the surface of the track. **4**

In dry conditions, $v = 2V$ and $\mu = \frac{3}{4}$. Use the previous results to find the value of $\tan\alpha$. **3**

In wet conditions, the coefficient of friction is reduced to a new value μ', such that a car at rest on the wet track is on the point of slipping **down** the banking. Find the value of μ'. **2**

Show that the maximum speed for taking the bend without skidding in wet conditions is approximately 82% of the maximum speed in dry conditions. **3**