Higher Mathematics

Supplementary Material

Expressions and Formulae

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I can convert radians to degrees and vice versa.

1. Convert the following angles from degrees to radians, giving your answer as an exact value.

(a)	$30°$	**(b)**	$45°$	**(c)**	$60°$
(d)	$90°$	**(e)**	$180°$	**(f)**	$360°$
(g)	$120°$	**(h)**	$135°$	**(i)**	$150°$
(j)	$210°$	**(k)**	$225°$	**(l)**	$240°$
(m)	$270°$	**(n)**	$300°$	**(o)**	$315°$
(n)	$330°$	**(o)**	$540°$	**(p)**	$720°$

2. Convert the following angles from degrees to radians, giving your answer to 3 significant figures.

(a)	$37°$	**(b)**	$142°$	**(c)**	$226°$
(d)	$281°$	**(e)**	$307°$	**(f)**	$453°$

3. Convert the following angles from radians to degrees.

(a)	π radians	**(b)**	2π radians	**(c)**	3π radians
(d)	$\frac{\pi}{2}$ radians	**(e)**	$\frac{3\pi}{2}$ radians	**(f)**	$\frac{5\pi}{2}$ radians
(g)	$\frac{\pi}{3}$ radians	**(h)**	$\frac{2\pi}{3}$ radians	**(i)**	$\frac{4\pi}{3}$ radians
(j)	$\frac{5\pi}{3}$ radians	**(k)**	$\frac{7\pi}{3}$ radians	**(l)**	$\frac{\pi}{4}$ radians
(m)	$\frac{3\pi}{4}$ radians	**(n)**	$\frac{5\pi}{4}$ radians	**(o)**	$\frac{7\pi}{4}$ radians
(p)	$\frac{\pi}{6}$ radians	**(q)**	$\frac{5\pi}{6}$ radians	**(r)**	$\frac{7\pi}{6}$ radians
(s)	$\frac{11\pi}{6}$ radians	**(t)**	$\frac{\pi}{12}$ radians	**(u)**	$\frac{5\pi}{12}$ radians

4. Convert the following angles from radians to degrees, giving your answer to 3 significant figures.

(a)	1 radian	**(b)**	2 radians	**(c)**	3 radians
(d)	4 radians	**(e)**	$1 \cdot 4$ radians	**(f)**	$2 \cdot 7$ radians

R2 I can use and apply exact values.

1. Write down the exact value of

(a)	$\sin 30°$	**(b)**	$\sin 60°$	**(c)**	$\sin 45°$
(d)	$\sin 120°$	**(e)**	$\sin 150°$	**(f)**	$\sin 135°$
(g)	$\sin 90°$	**(h)**	$\sin 180°$	**(i)**	$\sin 270°$
(j)	$\sin 210°$	**(k)**	$\sin 225°$	**(l)**	$\sin 240°$
(m)	$\sin 300°$	**(n)**	$\sin 330°$	**(o)**	$\sin 315°$

2. Write down the exact value of

(a) $\cos 30°$ **(b)** $\cos 60°$ **(c)** $\cos 45°$

(d) $\cos 120°$ **(e)** $\cos 150°$ **(f)** $\cos 135°$

(g) $\cos 90°$ **(h)** $\cos 180°$ **(i)** $\cos 270°$

(j) $\cos 210°$ **(k)** $\cos 225°$ **(l)** $\cos 240°$

(m) $\cos 300°$ **(n)** $\cos 330°$ **(o)** $\cos 315°$

3. Write down the exact value of

(a) $\tan 30°$ **(b)** $\tan 60°$ **(c)** $\tan 45°$

(d) $\tan 120°$ **(e)** $\tan 150°$ **(f)** $\tan 135°$

(g) $\tan 90°$ **(h)** $\tan 180°$ **(i)** $\tan 270°$

(j) $\tan 210°$ **(k)** $\tan 225°$ **(l)** $\tan 240°$

(m) $\tan 300°$ **(n)** $\tan 330°$ **(o)** $\tan 315°$

4. Write down the exact value of

(a) $\sin\frac{\pi}{6}$ **(b)** $\sin\frac{\pi}{4}$ **(c)** $\sin\frac{\pi}{3}$

(d) $\sin\pi$ **(e)** $\sin 2\pi$ **(f)** $\sin\frac{3\pi}{2}$

(g) $\sin\frac{5\pi}{6}$ **(h)** $\sin\frac{3\pi}{4}$ **(i)** $\sin\frac{2\pi}{3}$

(j) $\sin\frac{7\pi}{6}$ **(k)** $\sin\frac{5\pi}{4}$ **(l)** $\sin\frac{4\pi}{3}$

(m) $\sin\frac{11\pi}{6}$ **(n)** $\sin\frac{7\pi}{4}$ **(o)** $\sin\frac{5\pi}{3}$

5. Write down the exact value of

(a) $\cos\frac{\pi}{6}$ **(b)** $\cos\frac{\pi}{4}$ **(c)** $\cos\frac{\pi}{3}$

(d) $\cos\pi$ **(e)** $\cos 2\pi$ **(f)** $\cos\frac{3\pi}{2}$

(g) $\cos\frac{5\pi}{6}$ **(h)** $\cos\frac{3\pi}{4}$ **(i)** $\cos\frac{2\pi}{3}$

(j) $\cos\frac{7\pi}{6}$ (k) $\cos\frac{5\pi}{4}$ (l) $\cos\frac{4\pi}{3}$

(m) $\cos\frac{11\pi}{6}$ (n) $\cos\frac{7\pi}{4}$ (o) $\cos\frac{5\pi}{3}$

6. Write down the exact value of

(a) $\tan\frac{\pi}{6}$ (b) $\tan\frac{\pi}{4}$ (c) $\tan\frac{\pi}{3}$

(d) $\tan\pi$ (e) $\tan 2\pi$ (f) $\tan\frac{3\pi}{2}$

(g) $\tan\frac{5\pi}{6}$ (h) $\tan\frac{3\pi}{4}$ (i) $\tan\frac{2\pi}{3}$

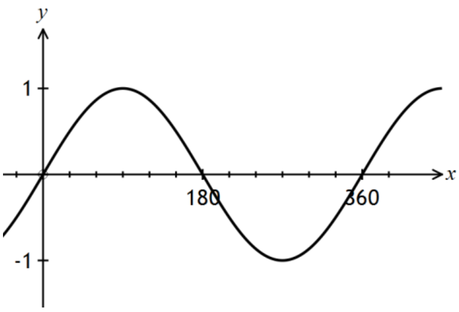
(j) $\tan\frac{7\pi}{6}$ (k) $\tan\frac{5\pi}{4}$ (l) $\tan\frac{4\pi}{3}$

(m) $\tan\frac{11\pi}{6}$ (n) $\tan\frac{7\pi}{4}$ (o) $\tan\frac{5\pi}{3}$

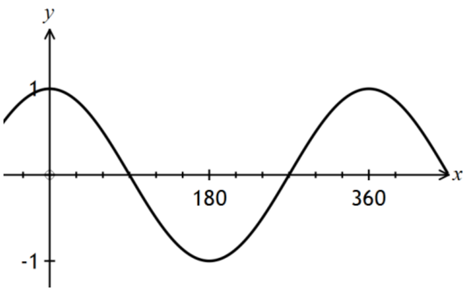
R3 I can sketch or identify a basic trig graph under a single transformation.

1. Write down the equation of each of the graphs

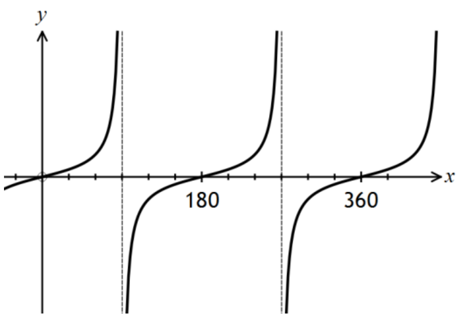
(a)



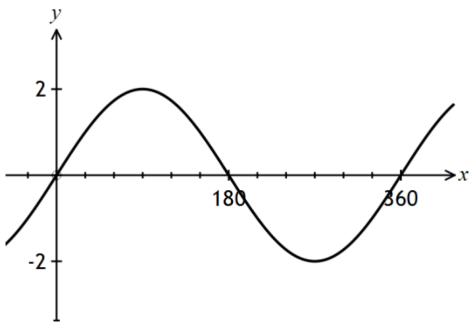
(b)



(c)

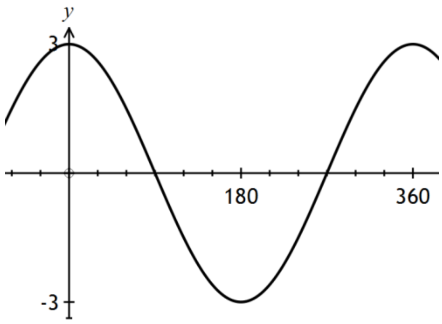


(d)

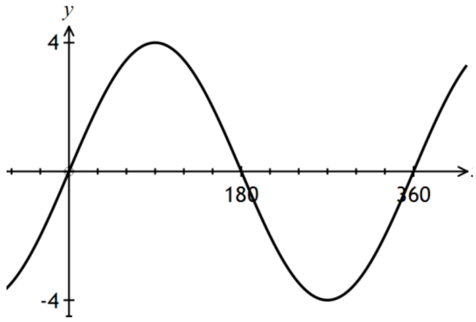


2. Write down the equation of each of the graphs

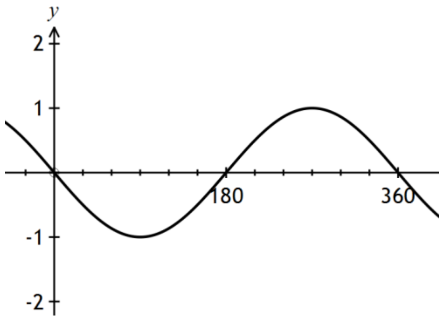
(a)



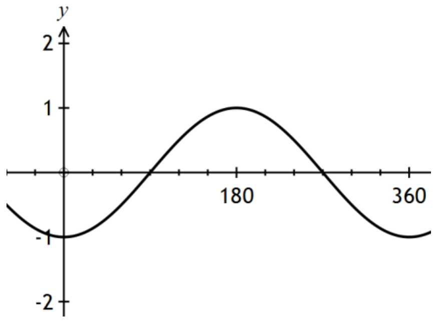
(b)



(c)

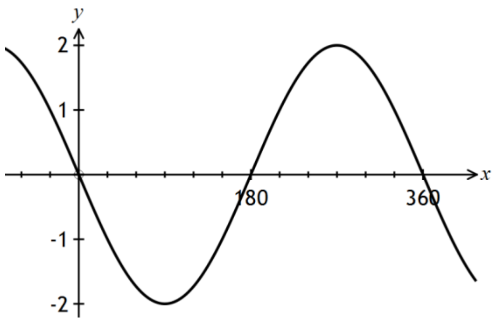


(d)

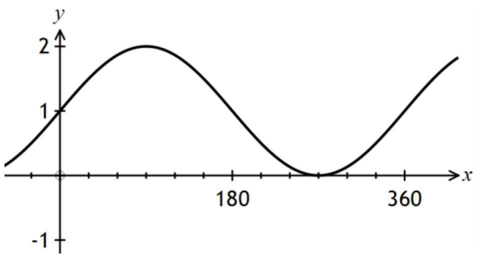


3. Write down the equation of each of the graphs

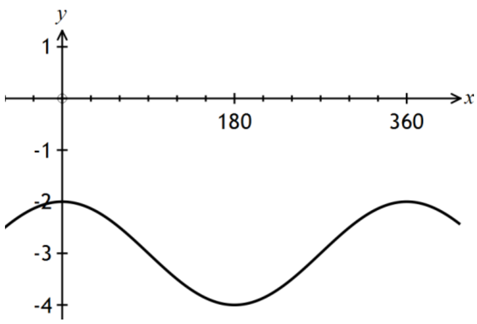
(a)



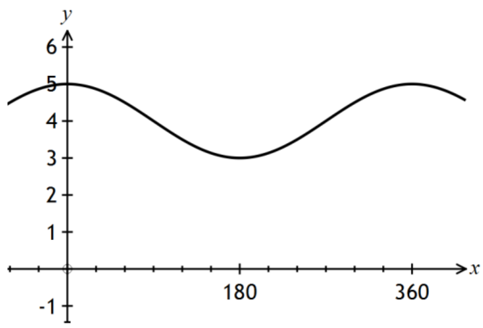
(b)



(c)

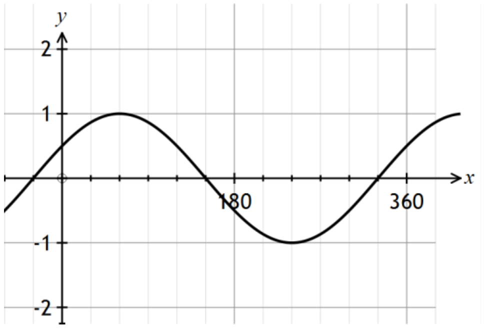


(d)

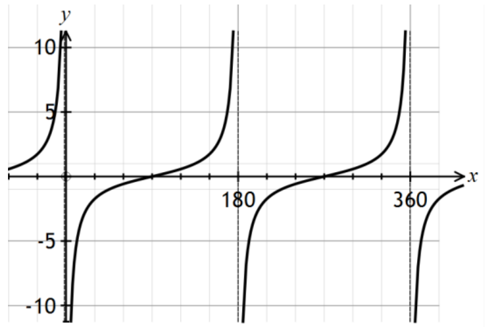


4. Write down the equation of each of the graphs

(a)



(b)



5. Sketch each graph showing clearly the coordinates of the maximum and minimum values and where each graph cuts the axes.

(a) $y = 3\cos x°$ for $0 \leq x \leq 360$

(b) $y = \sin x° + 1$ for $0 \leq x \leq 360$

(c) $y = \cos x - 1$ for $0 \leq x \leq 2\pi$

(d) $y = 2\sin x$ for $0 \leq x \leq 2\pi$

(e) $y = -2\cos x°$ for $0 \leq x \leq 360$

(f) $y = \tan(x - 45)°$ for $0 \leq x \leq 360$

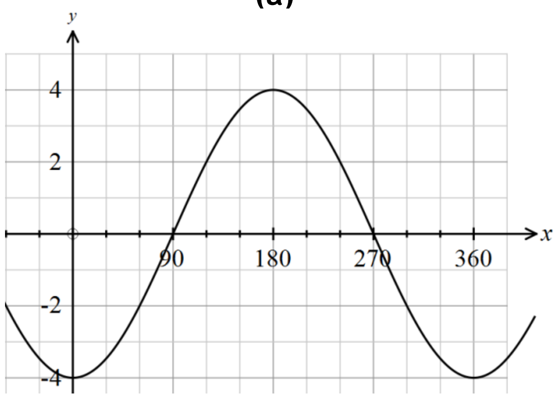
(g) $y = \cos\left(x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$

(h) $y = -3\sin x$ for $0 \leq x \leq 2\pi$

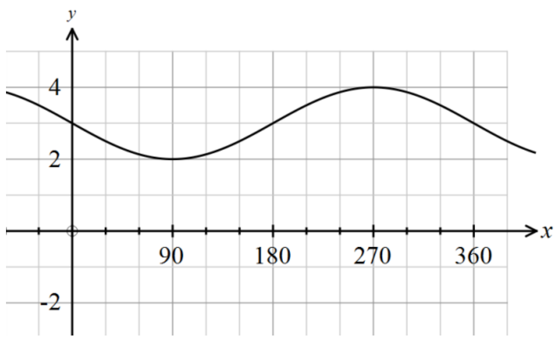
R4 I can sketch or identify a basic trig graph under combined transformations.

1. Write down the equation of each of the graphs

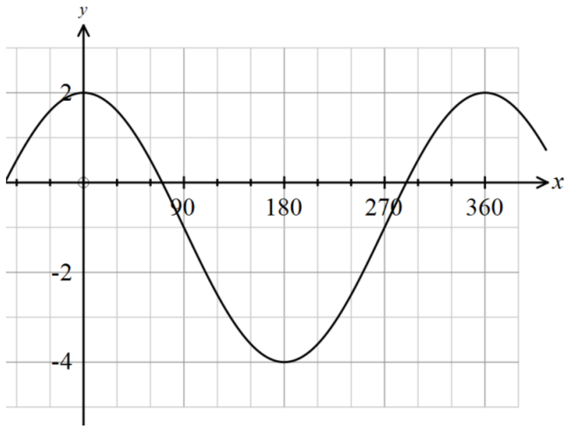
(a)



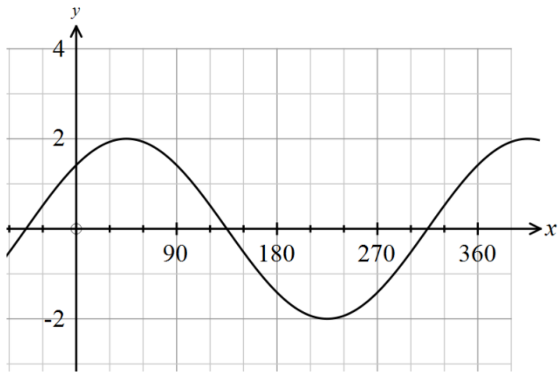
(b)



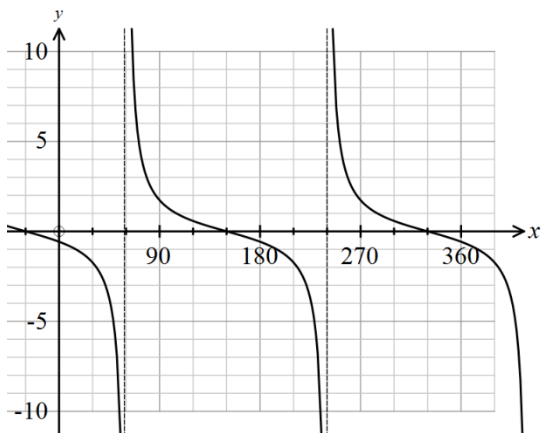
(c)


y
2
-2
-4
90
180
270
360
x

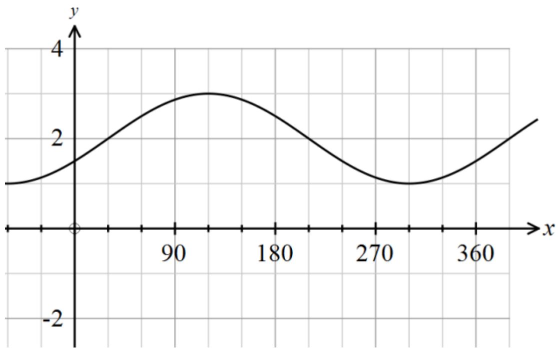
(d)


y
4
2
-2
90
180
270
360
x

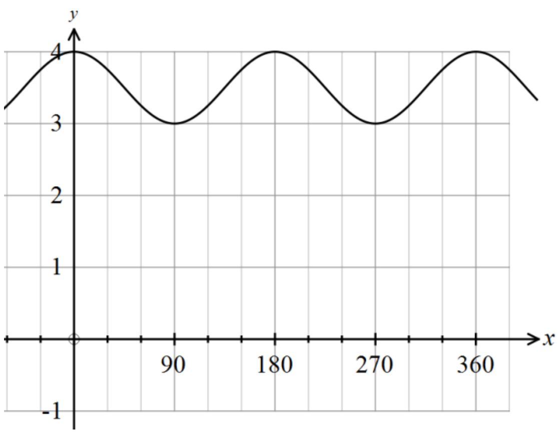
(e)


y
10
5
-5
-10
90
180
270
360
x

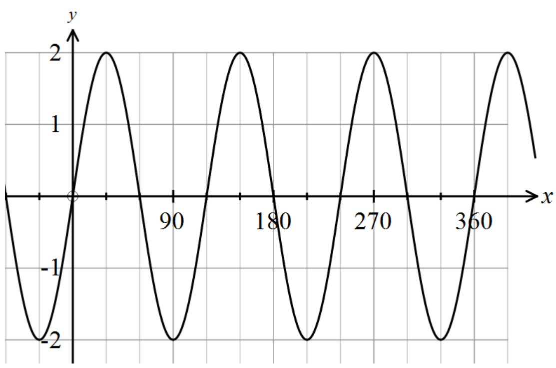
(f)


y
4
2
-2
90
180
270
360
x

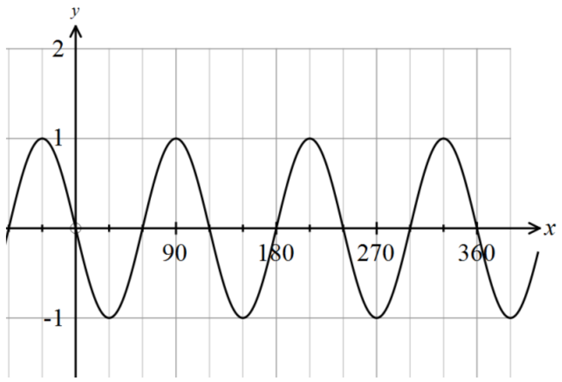
(g)


y
4
3
2
1
-1
90
180
270
360
x

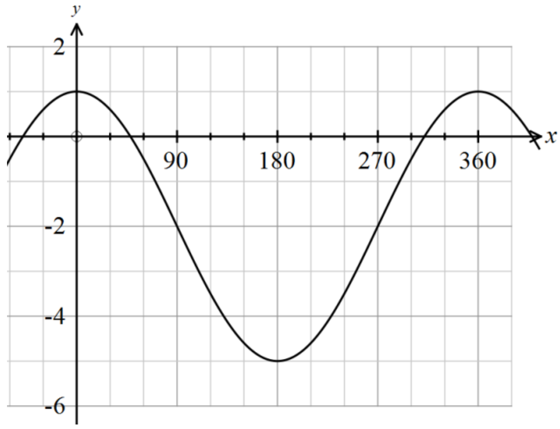
(h)


y
2
1
-1
-2
90
180
270
360
x

(i)



(j)



2. Sketch each graph showing clearly the coordinates of the maximum and minimum values and where each graph cuts the axes.

(a) $y = 2\cos x° - 3$ for $0 \leq x \leq 360$ **(b)** $y = -\cos 2x°$ for $0 \leq x \leq 360$

(c) $y = 2\cos\left(x - \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$

(d) $y = \sin\left(x - \frac{\pi}{6}\right) + 2$ for $0 \leq x \leq 2\pi$

(e) $y = 4\cos 2x°$ for $0 \leq x \leq 360$

(f) $y = -\sin(x - 30)°$ for $0 \leq x \leq 360$

(g) $y = \cos 2x - 1$ for $0 \leq x \leq 2\pi$ **(h)** $y = 2 - \sin x$ for $0 \leq x \leq 2\pi$

R5 I can use the addition and double angle formulae.

1. Expand and use exact values to simplify

(a) $\sin\left(x + \frac{\pi}{6}\right)$ **(b)** $\sin(x - 60)°$ **(c)** $\cos\left(x - \frac{\pi}{4}\right)$

(d) $\cos(x + 45)°$ **(e)** $\cos\left(x + \frac{\pi}{3}\right)$ **(f)** $\sin(x + 60)°$

(g) $\sin(x - 90)°$ **(h)** $\sin(x + \pi)$ **(i)** $\cos(x + 180)°$

2. Use an appropriate substitution (such as 45 - 30 = 15) then expand to find the exact values of

(a) $\sin 15°$ **(b)** $\sin 75°$ **(c)** $\cos 105°$

3. Given that $\sin x° = \frac{3}{5}$ and $\cos x° = \frac{4}{5}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\sin 3x°$ (Hint $3x = 2x + x$)

4. Given that $\sin x° = \frac{5}{13}$ and $\cos x° = \frac{12}{13}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\sin 4x°$ (Hint $4x = 2(2x)$)

5. Given that $\sin x° = \frac{1}{\sqrt{5}}$ and $\cos x° = \frac{2}{\sqrt{5}}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\cos 3x°$

6. Given that $\sin x° = \frac{2}{\sqrt{13}}$ and $\cos x° = \frac{3}{\sqrt{13}}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\cos 4x°$

R6	**I can convert $acosx + bsinx$ to $kcos(x \pm \alpha)$ or $ksin(x \pm \alpha)$, where α is in any quadrant $k > 0$.**

1. A function f is defined as $f(x) = 5\cos x° - 2\sin x°$.

Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

2. Express $\sin x - \cos x$ in the form $k\sin(x - \alpha)$ where $k > 0$ and $0 \leq \alpha < 2\pi$.

3. A function g is defined as $g(x) = 3\cos x° + \sin x°$.

Express $g(x)$ in the form $k\sin(x + \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$.

4. Express $\sin x + 2\cos x$ in the form $r\cos(x - a)$ where $r > 0$ and $0 \leq a < 2\pi$.

5. A function Q is defined as $Q(x) = 2\cos x° - 3\sin x°$.

Express $Q(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

6. Express $3\sin x - 4\cos x$ in the form $a\sin(x - b)$ where $a > 0$ and $0 \leq b < 2\pi$.

7. A function f is defined as $f(x) = 2\cos x° - \sin x°$.

Express $f(x)$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

8. Express $\sin x - \sqrt{3}\cos x$ in the form $k\cos(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

9. A function f is defined as $f(x) = \sqrt{5}\cos x° + 3\sin x°$.

Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

R7 I have revised solving basic trigonometric equations in degrees and radians.

1. Solve the equations:

(a) $5\tan x° - 6 = 2, \quad 0 \leq x \leq 360.$

(b) $7\sin x° + 1 = -5, \quad 0 \leq x \leq 360.$

(c) $4\cos x° + 3 = 0, \quad 0 \leq x \leq 360.$

(d) $3\tan x + 3 = 7, \quad 0 \leq x \leq 2\pi.$

(e) $4\sin x - 2 = -3, \quad 0 \leq x \leq 2\pi.$

(f) $9\cos x - 5 = 0, \quad 0 \leq x \leq 2\pi.$

2. Solve the equations:

(a) $9\tan 2x° - 5 = 3, \quad 0 \leq x \leq 180.$

(b) $4\sin 3x° + 1 = -2, \quad 0 \leq x \leq 360.$

(c) $3\cos 2x° + 2 = 0, \quad 0 \leq x \leq 360.$

3. Solve the equations:

(a) $\tan(x + 30)° = 3, \quad 0 \leq x \leq 360.$

(b) $5\sin(x + 10)° + 3 = -1, \quad 0 \leq x \leq 360.$

(c) $4\cos(x + 26)° + 3 = 0, \quad 0 \leq x \leq 360.$

(d) $\sqrt{3}\tan\left(x + \frac{\pi}{5}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi.$

(e) $6\sin(x + 2) - 2 = 1, \quad 0 \leq x \leq 2\pi.$

(f) $\sqrt{2}\cos\left(x + \frac{\pi}{6}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi.$

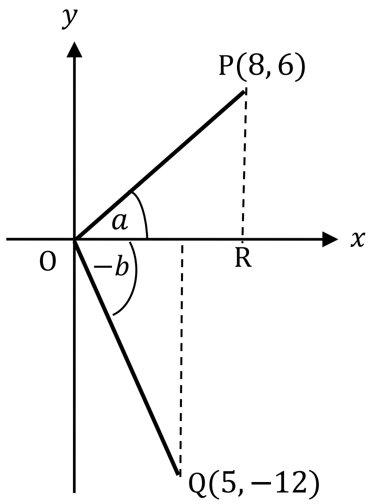
Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1	I can apply Trig Formulae to Mathematical Problems (excluding where trig equations have to be solved but including exact values).

1. On the coordinate diagram shown, P is the point $(8, 6)$ and Q is the point $(5, -12)$.

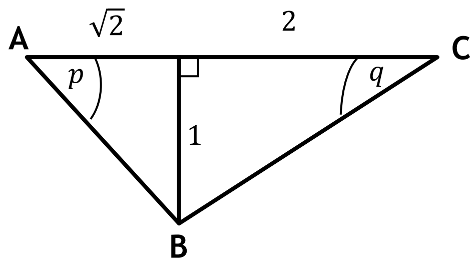
Angle POR = a and angle ROQ = $-b$.



(a) Find the exact value of $\sin(a - b)$.

(b) Find the exact value of $\cos 2a$.

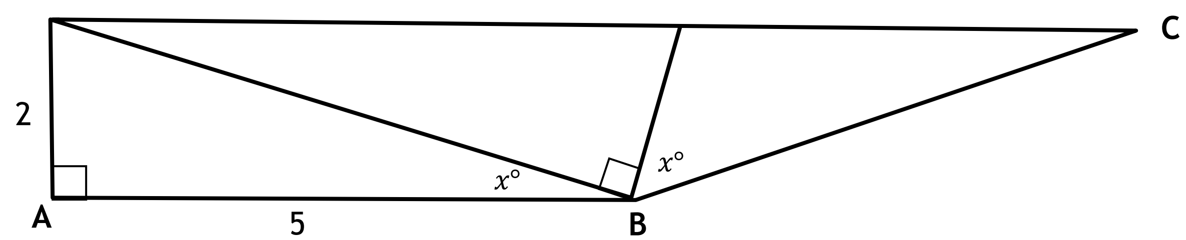
2. In triangle ABC, show that:



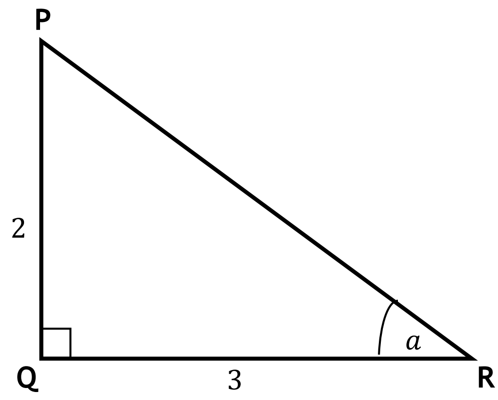
(a) The exact value of $\sin 2p = \frac{2\sqrt{2}}{3}$

(b) The exact value of $\cos(p + q) = \frac{2\sqrt{2}-1}{\sqrt{15}}$

3. For the shape shown, find the exact value of $\cos(A\hat{B}C)°$



4. The diagram shows the right angled triangle PQR, with dimensions given.



(a) Find the exact value of $\sin 2a°$.

(b) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$.

5. If $\cos 2x = -\frac{31}{49}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$.

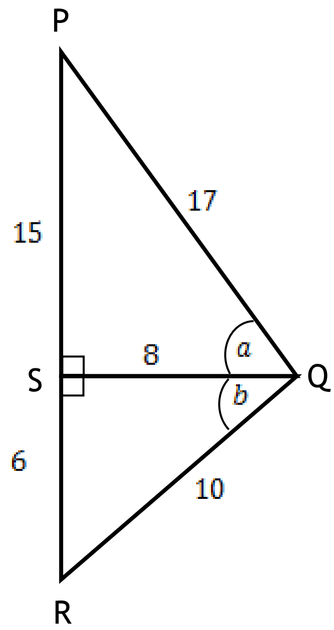
6. Using the fact that $\frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$, find the exact value of $\cos\left(\frac{5\pi}{12}\right)$.

7. It is given that $\cos a = \frac{3}{5}$ and $\sin b = \frac{2}{3}$.

(a) Find the exact value of $\sin(a + b)$ and $\cos(a + b)$.

(b) Hence find the exact value of $\tan(a + b)$.

8. Triangles PSQ and RSQ are right angled with dimensions as shown in the diagram.



(a) Show that $\cos(a + b)$ is $-\frac{13}{85}$.

(b) Calculate the value of $\sin(a + b)$.

(c) Hence calculate the value of $\tan(a + b)$.

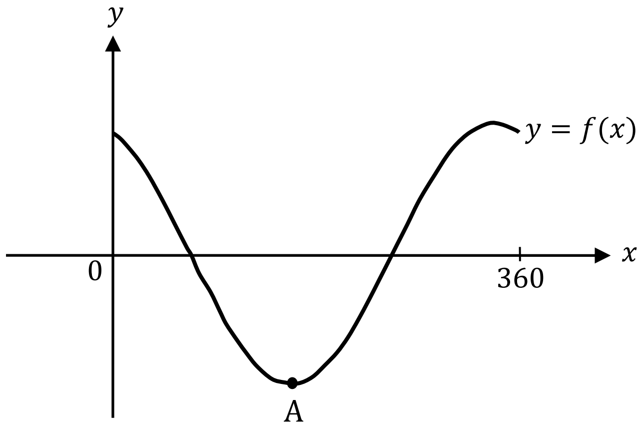
NR2	**I have experience of using wave functions to find the maximum and minimum values.**

1. A function f is defined as $f(x) = 5\cos x° - 2\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \le a < 360$.

(b) Part of the graph of $y = f(x)$ is shown in the diagram.

Find the coordinates of the minimum turning point A.



2. **(a)** Express $\sin x - \cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \le a < 2\pi$.

(b) Hence state the maximum and minimum values of $\sin x - \cos x$ and determine the values of x, in the interval $0 \le x < 2\pi$, at which these maximum and minimum values occur.

3. **(a)** Express $12\sin x + 5\cos x$ in the form $k\sin(x + a)$ where $k > 0$ and $0 \le a < 2\pi$.

(b) Hence state the maximum value of $4 + 12\sin x + 5\cos x$ and determine the value of x, in the interval $0 \le x < 2\pi$, at which the maximum occurs.

4. A function f is defined as $f(x) = 4\cos x° + 3\sin x°$.

Find the maximum and minimum values of $f(x)$ and the values of, in the range $0 \le x < 360$, at which the maximum and minimum values occur.

5. A function g is defined as $g(x) = 3\cos x - 2\sin x$.

Find the maximum and minimum values of $g(x)$ and the values of, in the range $0 \le x < 2\pi$, at which the maximum and minimum values occur.

Trigonometry, functions and graphs

1. A function f is defined as $f(x) = \sqrt{3}\cos x° + \sin x°$.

(a) Express $f(x)$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Sketch the graph of $y = f(x)$ between $0 \leq x < 360$, showing clearly the coordinates of the maximum and minimum turning points.

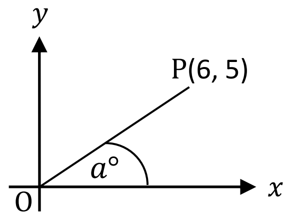
2. **(a)** Express $3\sin x° + 4\cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Sketch the graph of $y = 3\sin x° + 4\cos x° + 1$ between $0 \leq x < 360$, showing clearly the coordinates of the maximum and minimum turning points and where the curve cuts the axes.

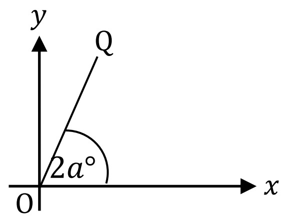
Trigonometry and straight line

1. P is the point $(6, 5)$. The line OP is inclined at an angle of $a°$ to the x-axis.

(a) Find the exact values of $\sin 2a°$ and $\cos 2a°$.



(b) The line OQ is inclined at an angle of $2a°$ to the x-axis. Write down the exact value of the gradient of OQ.



Logs and Exponentials
Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have had experience of simplifying expressions with surds and indices.

1. Simplify the following writing the answers with positive indices only.

(a) $x^2 \times x^7$ (b) $y^{-3} \times y^8$ (c) $x^6 \div x^4$

(d) $y^{-3} \div y^{-1}$ (e) $(a^4)^3$ (f) $(p^{-4})^{-2}$

2. (a) $4x^3 \times 2x^2$ (b) $5x^3 \times 4x^{-3}$ (c) $\dfrac{3x^5y^3}{6x^2y^5}$

(d) $\dfrac{4r^8}{2r^{-5}}$ (e) $\sqrt{x} \times x^2$ (f) $\sqrt{x} \times \sqrt[3]{x^2}$

(g) $\frac{1}{\sqrt{a}} \times \sqrt{a^3}$ (h) $\sqrt[3]{x} \times x^{-\frac{1}{3}}$ (i) $x^{-2}(x^2+1)$

R2 I can write an exponential in logarithmic form and vice versa.

1. For each exponential relationship, write a related logarithmic relationship.

(a) $3^x = 5$ (b) $8^t = 100$ (c) $5^r = 13$

(d) $10^r = 5$ (e) $6^s = 15$ (f) $2^p = 32$

(g) $a^x = 5$ (h) $b^y = 5$ (i) $c^z = 5$

(j) $2^3 = 8$ (k) $3^2 = 9$ (l) $10^4 = 10000$

(m) $a^b = c$ (n) $x^y = z$ (o) $p^q = r$

2. For each logarithmic relationship, write a related exponential relationship.

(a) $\log_x 20 = 3$ (b) $\log_e x = 2$ (c) $\log_3 7 = x$

(d) $\log_y 16 = 4$ (e) $\log_e r = 1 \cdot 2$ (f) $\log_6 9 = t$

(g) $\log_3 x = 2$ **(h)** $\log_{10} y = 2 \cdot 4$ **(i)** $\log_{(e+1)} R = v$

(j) $\log_{(x+2)} T = r$ **(k)** $\log_5 r = x + 1$ **(l)** $\log_e x = r - 2$

R3 I can solve exponential equations using logarithms.

1. Solve each of the following exponential equations

(a) $10^x = 1000$ **(b)** $10^x = 0 \cdot 01$ **(c)** $10^x = 100000$

(d) $10^x = 20$ **(e)** $10^x = 3000$ **(f)** $10^x = 0 \cdot 05$

(g) $10^{2p} = 5$ **(h)** $10^{9t} = 500$ **(i)** $10^{-4r} = 0 \cdot 6$

(j) $10^{-3p} = 20$ **(k)** $10^{0 \cdot 1y} = 3000$ **(l)** $10^{0 \cdot 5q} = 0 \cdot 05$

2. Solve each of the following exponential equations

(a) $e^x = 7$ **(b)** $e^x = 23$ **(c)** $e^{2t} = 9$

(d) $e^{3p} = 16$ **(e)** $e^{-3x} = 0 \cdot 4$ **(f)** $e^{-9r} = 1 \cdot 3$

(g) $6e^x = 12$ **(h)** $4e^{0 \cdot 02x} = 12$ **(i)** $12e^{3t} = 6$

(j) $3e^{-4p} = 21$ **(k)** $2e^{-2x} = 0 \cdot 4$ **(l)** $6e^{-0 \cdot 5r} = 1 \cdot 3$

3. Solve each of the following exponential equations

(a) $4 \times 10^x = 400$ **(b)** $7 \times 10^p = 0 \cdot 67$ **(c)** $3 \times 10^t = 12$

(d) $0 \cdot 5 \times 10^a = 20$ **(e)** $\frac{10^b}{8} = 12 \cdot 5$ **(f)** $\frac{10^c}{100} = 0 \cdot 05$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can manipulate logarithms and exponentials and apply the three main laws of logarithms.

1. Given $b = e^t$ which of the following is true:

(a) $\log_t b = e$

(b) $\log_e b = t$

2. Given $\log_n x = y$ which of the following is true:

(a) $n^y = x$

(b) $x^y = n$

3. Simplify

(a) $\log_x 3 + \log_x 5 - \log_x 7$

(b) $\log_a 32 - 2\log_a 4$

4. Show that **(a)** $\dfrac{\log_3 8}{\log_3 2} = 3$. **(b)** $\dfrac{\log_b 9a^2}{\log_b 3a} = 2$.

5. If $\log_3 x = 2\log_3 y - 3\log_3 z$ find an expression for *x* in terms of *y* and *z*.

6. Find a if $\log_a 64 = \frac{3}{2}$.

7. Simplify $2\log_e(3e) - 4\log_e(2e)$ expressing your answer in the form $\log_e B - \log_e C - A$ where A, B and C are whole numbers.

NR2 I can solve exponentials and logarithmic equations using the laws of logarithms.

1. Given the equation $y = 3 \times 4^x$ find the value of x when $y = 10$ giving your answer to 3 significant figures.

2. Given the equation $A = A_0 e^{-kt}$, find, to 3 significant figures:

(a) A when $A_0 = 5$, $k = 0 \cdot 23$ and $t = 20$.

(b) k when $A = 70$, $A_0 = 35$ and $t = 20$.

(c) t when $A = 1000$, $A_0 = 10$ and $k = -0.01$.

3. **(a)** Solve the equation $\log_3(3 - 2x) + \log_3(2 + x) = 1$.

(b) Solve $\log_3(5 - x) - \log_3(3 - x) = 2,\ x < 3$.

(c) Solve $\log_4 x + \log_4(x + 6) = 2$.

4. **(a)** Given that $\log_5 x = A$, show that $\log_{25} x = \frac{1}{2}A$.

(b) Solve $\log_4 x + \log_{16} x = 3$.

5. The curve with equation $y = \log_4(x - 1) - 2$, where $x > 1$, cuts the x-axis at the point $(p, 0)$. Find the value of p.

6. If $\log_4 8 + \log_4 q = 1$, find the value of q.

7. Solve the equation $\log_3(x + 2) - 3\log_3 2 = 2$.

8. Find x if $4\log_x 6 - 2\log_x 4 = 1$.

9. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x - 2) + 1$ intersects the x-axis.

NR3	**I have experience of plotting and extracting information from straight line graphs with logarithmic axes (axis).**

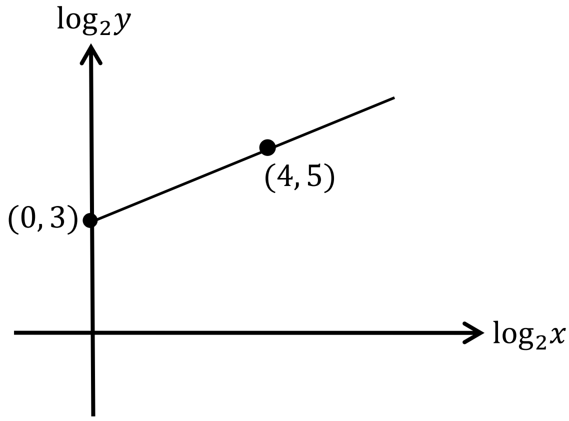
1. Show that $y = kx^n$, where k and n are constants, can be expressed as a straight line in terms of $\log y$ and $\log x$.

2. Show that $y = Ae^{kx}$, where k and n are constants, can be expressed as a straight line in terms of $\log y$ and x.

3. Variables x and y are related by the equation $y = kx^n$

The graph of $log_2 y$ against $log_2 x$ is a straight line through the points $(0, 3)$ and $(4, 5)$, as shown in the diagram.

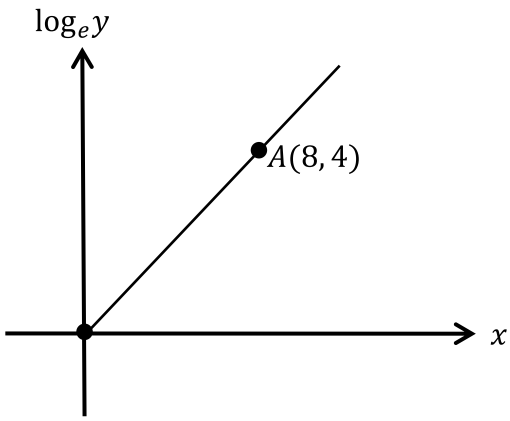
Find the values of k and n.



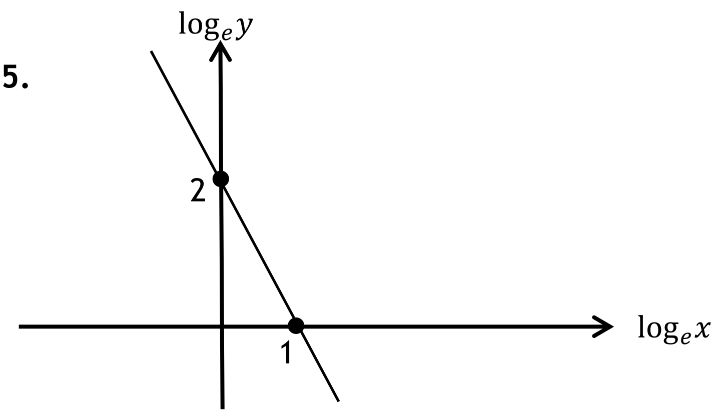
4. Two variables, x and y, are connected by the law $y = a^x$.

The graph of $\log_e y$ against x is a straight line passing through the origin and the point $A(8, 4)$.

Find the value of a.



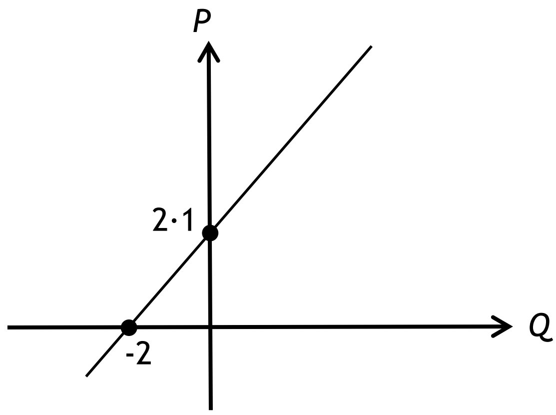
5.



The graph illustrates the law $y = kx^n$.

If the straight line passes through $A(1, 0)$ and $B(0, 2)$, find the values of k and n.

6. The result of an experiment gives rise to the graph shown.



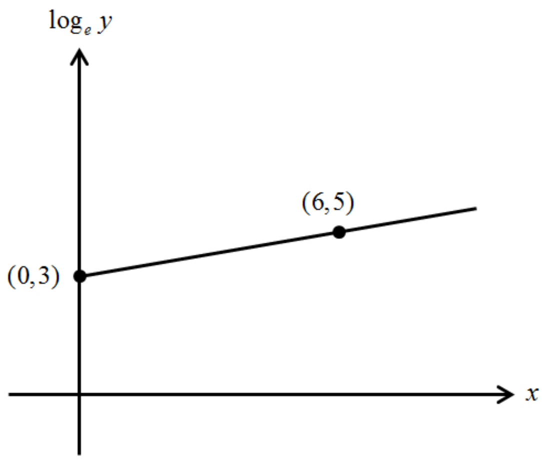
(a) Write down the equation of the line in terms of P and Q.

(b) If $P = \log_{10}p$ and $Q = \log_{10}q$, show that p and q satisfy $p = aq^b$ stating the values of a and b.

7. Variables x and y are related by the equation $y = Ae^{kx}$

The graph of $\log_e y$ against x is a straight line through the points $(0, 3)$ and $(6, 5)$, as shown in the diagram.

Find the values of k and A.



8. Two variables x and y satisfy the equation $y = 5 \times 4^x$.

(a) Find the value of a if $(a, 10)$ lies on the graph with equation $y = 5 \times 4^x$.

(b) If $(-\frac{1}{2}, b)$ also lies on the graph, find b.

(c) A graph is drawn of $\log_{10}y$ against x. Show that its equations will be of the form $log_{10}y = Px + Q$ and state the gradient of this line.

NR4 I can solve logarithmic and exponential equations in real life contexts.

1. The amount of a certain radioactive element, A_t , remaining after t years can be found using the formula $A_t = A_0e^{-0\cdot002t}$, where A_0 is the amount present initially.

(a) If 300 grams are left after 500 years, how many grams were present initially.

(b) The half-life of a substance is the time taken for the amount to decrease to half its initial amount. What is the half life of this substance?

2. The value V (in £ thousand) of a car is shown to depreciate after t years from first purchase according to the formula $\mathrm{V} = 18\mathrm{e}^{-0\cdot15\mathrm{t}}$.

(a) What was the value of the car when first purchased?

(b) The car was sold when its value had dropped to 10% of the value when first purchased.

After how many years was the car sold?

3. The formula $A_t = A_0e^{-0\cdot000124t}$ is used to determine the age of wood, where A_0 is the amount of carbon-14 in any living tree, A_t is the amount of carbon-14 in the wood being dated and t is the age of the wood in years.

A wooden artefact was found to contain 90% of the carbon-14 of a living tree.

Is the artefact over 500 years old?

4. The size of a rabbit population, N, can be modelled using the equation $N = N_0 e^{kt}$ where N_0 is the population at the beginning of a study and t is the time in years since the study began and k is a constant.

(a) The rabbit population comprised of 70 individuals at the beginning of the study. If $k = 0.05$ find the size of the rabbit population after six years.

(b) How long will it take the rabbit population to double in size?

5. Radium decays exponentially and its half-life is 1600 years.

If A_0 represents the amount of radium in a sample to start with and $A(t)$ represents the amount remaining after t years, then $(t) = A_0 e^{-kt}$.

(a) Determine the value of k, correct to 3 significant figures.

(b) Hence find what percentage, to the nearest whole number, of the original amount of radium will be remaining after 2500 years.

6. The concentration of a fertiliser in the soil can be modelled by the equation $F = F_0 e^{-kt}$ where F_0 is the initial concentration, F_t is the concentration at time t and t is the time, in days, after the application of the pesticide.

(a) If it takes 20 days for the level of the fertiliser in the soil to reduce by 25%, find the value of k to 2 significant figures.

(b) Eighty days after the initial application, what is the percentage decrease in the concentration of the fertiliser?

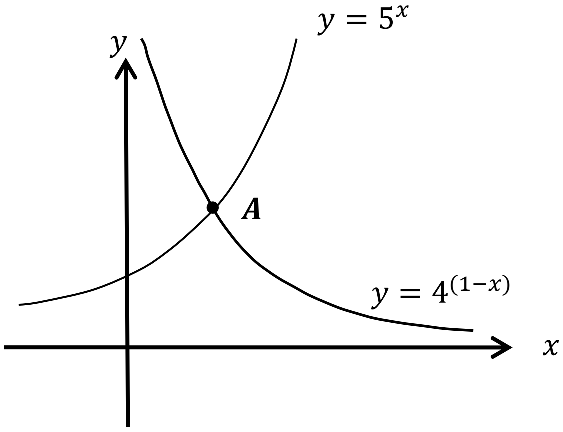
7. The spread of disease in trees was described by a law of the form $A(t) = A_0 e^{kt}$ where A_0 is the area covered by the disease when it was first detected and A is the area covered by the disease t months later.

If it takes six months for the area of the disease to double, find the value of the constant, k, correct to 3 significant figures.

NR5	I can display on, and extract information from, logarithmic and exponential graphs.

1. The diagram shows the curves with equations $y = 5^x$ and $y = 4^{(1-x)}$.

The graphs intersect at the point A.



(a) Show that the x-coordinate of A can be written in the form $\frac{\log_a C}{\log_a D}$, for all $a > 1$.

(b) Calculate the y-coordinate of A.

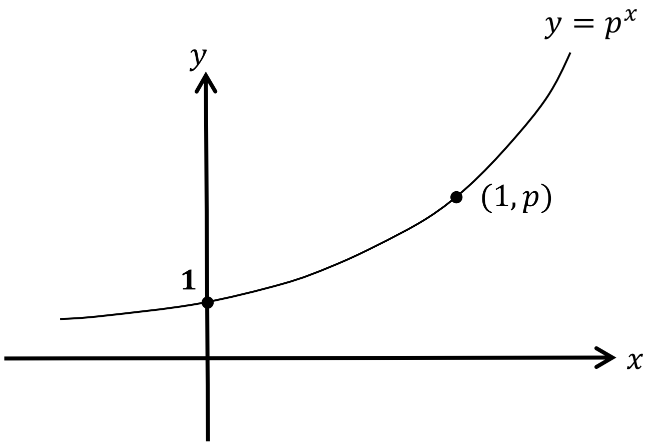
2. The diagram shows the graph of $y = p^x$, $p > 1$.

On separate diagrams sketch:

(a) $y = -p^x$

(b) $y = p^{-x}$

(c) $y = p^{2-x}$



3. Sketch the graph of $y = \log_a x$. On the same diagram sketch:

(a) $y = \log_a\left(\frac{1}{x}\right)$

(b) $y = \log_a(x - 3)$

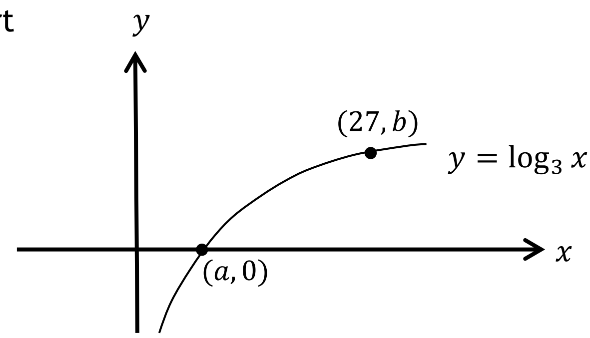
(c) $y = \log_a x + 2$

4. Sketch the graph of $y = p^x + 1$.

(a) On the same diagram sketch the graph of $y = p^{x+1}$.

(b) Prove that the two graphs intersect at a point where the x-coordinate is $\log_p\left(\frac{1}{p-1}\right)$.

5. The diagram shows a sketch of part of the graph of $y = \log_3 x$

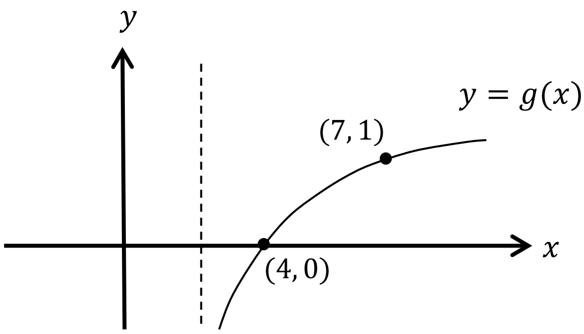


(a) Write down the values of a and b.

(b) Sketch the graph of $y = \log_3(x - 1) + 2$

6. The function g is of the form $g(x) = \log_p(x - q)$.

The graph of $y = g(x)$ is shown in the diagram.



(a) Write down the values of p and q.

(b) State the domain of g.

NR6 I have experience of cross topic exam standard questions

Logarithms with composite functions

1. Functions p, q and r are defined on suitable domains by

$$p(x) = x^2 - 12x + 19,\ q(x) = 5 - x \text{ and } r(x) = \log_2 x.$$

(a) Find expressions for $r(p(x))$ and $r(q(x))$.

(b) Hence solve $r\big(p(x)\big) - r\big(q(x)\big) = 3$.

Logarithms with polynomials

2. **(a)** Show that $x = -4$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

Hence factorise $x^3 + 8x^2 + 11x - 20$ fully.

(b) Solve $\log_2(x+3) + \log_2(x^2 + 5x - 4) = 3$.

Functions and Graphs

Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have had experience of graphing linear and quadratic functions.

1. Sketch the graphs of the following straight lines:

(a) $y = 2x + 3$ **(b)** $y = -3x - 2$ **(c)** $y = \frac{1}{2}x + 1$

(d) $2x + y - 4 = 0$ **(e)** $x - 3y + 6 = 0$ **(f)** $3x + 4y - 8 = 0$

2. For the following Quadratic Functions:

- Calculate where the graph crosses the x-axis and the y-axis
- Find the Turning Point and state it's nature
- Sketch the graph

(a) $y = x^2 - 4x + 3$ **(b)** $y = x^2 + 10x + 24$ **(c)** $y = x^2 + 2x - 15$

(d) $y = x^2 - 4x - 12$ **(e)** $y = x^2 - x - 12$ **(f)** $y = 4x^2 - 8x + 3$

3. For the following Quadratic Functions:

- Express in the form $y = a(x + b)^2 + c$
- State the Turning Point and state it's nature
- Sketch the graph indicating the turning point and y-intercept

(a) $y = x^2 - 4x + 5$ **(b)** $y = x^2 + 6x - 1$ **(c)** $y = x^2 - 3x + 4$

(d) $y = 2x^2 - 12x + 5$ **(e)** $y = 2 + 8x - x^2$ **(f)** $y = 3x^2 - 12x - 4$

(g) $y = 3 + 6x - x^2$ **(h)** $y = 5 - 12x - 2x^2$ **(i)** $y = x^2 + 5x - 2$

(j) $y = 3x^2 - 18x + 5$ **(k)** $y = 2x^2 + 8x - 4$ **(l)** $y = 4x^2 - 8x + 1$

(m) $y = 2x^2 - 10x - 3$ **(n)** $y = 2 + 8x - 2x^2$ **(o)** $y = 4x^2 - 16x + 9$

(p) $y = 7 + 12x - 3x^2$ **(q)** $y = 5 - 12x - 4x^2$ **(r)** $y = 6x^2 + 24x - 5$

R2 I have found x and y - intercepts for a range of graphs of functions.

1. Calculate where the graph of the following functions crosses the x-axis and the y-axis:

(a) $y = 4x + 8$ **(b)** $y = \frac{1}{4}x - 3$ **(c)** $3x + 5y - 15 = 0$

(d) $y = x^2 - 3x$ **(e)** $y = x^2 + 9x$ **(f)** $y = 3x^2 - 12x$

(g) $y = x^2 - 16$ **(h)** $y = 4x^2 - 9$ **(i)** $y = 2x^2 - 18$

(j) $y = x^2 - 7x + 10$ **(k)** $y = x^2 + 6x - 27$ **(l)** $y = 6x^2 - 13x - 5$

(m) $y = \sqrt{x + 4}$ **(n)** $y = \sqrt{2x + 9}$ **(o)** $y = \sqrt{x + 16}$

R3 I can solve linear and quadratic inequalities

1. Solve the following inequalities:

(a) $4x - 12 < 0$ **(b)** $15 - 3x \geq 0$ **(c)** $6x + 15 \leq 0$

(d) $-2x - 7 > 0$ **(e)** $5x + 17 \leq 0$ **(f)** $18 - 4x \geq 0$

(g) $x^2 - 5x = 0$ **(h)** $x^2 + 7x = 0$ **(i)** $4x^2 - 36x = 0$

(j) $x^2 - 49 = 0$ **(k)** $9x^2 - 25 = 0$ **(l)** $3x^2 - 12 = 0$

(m) $x^2 - 5x + 6 = 0$ **(n)** $x^2 + 8x - 20 = 0$ **(o)** $x^2 - 2x - 35 = 0$

(p) $6x^2 - 11x + 3 = 0$ **(q)** $2x^2 + 7x + 6 = 0$ **(r)** $4x^2 - 17x - 15 = 0$

R4 I can understand and use basic set notation.

1. Using the { } brackets notation, list the following sets:

(a) The set of months ending in the letter y.

(b) The set of the first ten prime numbers.

(c) The set of letters of the alphabet between G and P.

(d) The set of odd numbers greater than 20 but less than 30.

2. Describe the following sets in words:

(a) { January, June, July }

(b) { Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto }

(c) { Cone, Pyramid }

(d) { 1, 4, 9, 16, 25 }

3. Connect these elements with their sets, using $\in$:

Elements: a, green, 12, b, 13

Sets: {a,b,c}, {even numbers}, {colours}, {10,11,12,13}, {vowels in the English alphabet}, {red,yellow,green}

4. State which of the following are true and which are false:

(a) $2 \in$ { prime numbers }

(b) { x,y,z } and { p,q,r } are equal sets

(c) { 0 } is the empty set

(d) { k,l,m,n } = { m,l,k,n }

(e) If A = { whole numbers greater than 50 }, then $46 \notin A$

5. Using set notation, rewrite the following:

(a) 3 is a member of the set W.

(b) The empty set.

(c) x does not belong to the set A.

(d) S is a subset of the set T.

(e) The set P is equal to the set Q.

6. S = { 1,2,3,4,5,6,7,8,9,10 }. List the following subsets of S:

(a) The set of prime numbers in S.

(b) The set of odd numbers in S less than 8.

(c) The set of elements in S which are factors of 70.

(d) The set of numbers in S which are divisible by 3.

7. Find a set equal to each of the following:

(a) $\{ 1,2,3 \} \cap \{ 2,3,4,5 \}$

(b) $\{ 1,2,3 \} \cap \{ 4,5,6 \}$

(c) $\{ 1,2,3 \} \cap \{ 3,1,2 \}$

(d) $\emptyset \cap \{ 2,3,4,5 \}$

8. E = { 1,2,3,4,5,6,8,10 } A = { 1,2,3,4 } B = { 3,4,5 } and C = { 2,4,6,8,10 }

(a) Find $A \cap B$, $B \cap C$ and $A \cap C$.

(b) The set of elements common to A,B and C is denoted by $A \cap B \cap C$.

Find $A \cap B \cap C$.

9. Given that A = { 0,1,2 }, which of the following are true?

(a) $2 \in A$ **(b)** $1 \subset A$ **(c)** $\{1\} \subset A$

(d) $0 \in \emptyset$ **(e)** $A \subset A$ **(f)** $1 \notin A$

10. P = { 1,2,3,4,5,6,7 } Q = { 5,6,7,8,9,10 } are subsets of E = { 1,2,3,...,12 }.

List the members of the following sets:

(a) P ∩ Q	**(b)** P ∪ Q	**(c)** P’
(d) Q’	**(e)** (P ∩ Q)’	**(f)** (P ∪ Q)’
(g) P ∩ Q’	**(h)** P’ ∩ Q	**(i)** P ∩ ∅

R5 I have investigated domains and ranges.

1. State a suitable domain for the following functions:

(a) $f(x) = \frac{x^2}{x-1}$	**(b)** $f(x) = \frac{4x-2}{2x-3}$	**(c)** $f(x) = \frac{x^2+5}{(x-1)(x+4)}$
(d) $f(x) = \frac{4x^2}{x^2-3x}$	**(e)** $f(x) = \frac{2x+7}{x^2-16}$	**(f)** $f(x) = \frac{x^2-5x+4}{x^2+8x+12}$
(g) $f(x) = \sqrt{x-7}$	**(h)** $f(x) = \sqrt{10-x}$	**(i)** $f(x) = \sqrt{x^2-9}$
(j) $f(x) = \sqrt{36-x^2}$	**(k)** $f(x) = \sqrt{x^2-5x-6}$	**(l)** $f(x) = \sqrt{x^2+3x}$

2. State the range of each function given its domain:

(a) $f(x) = 3x - 4$; $x \in \{2, 3, 4, 5\}$

(b) $f(x) = x^2 - 3x + 4$; $x \in \{-2, -1, 0, 1, 2\}$

(c) $f(x) = 3x^2 - 7$; $x \in \{-3, -2, 0, 2, 3\}$

(d) $f(x) = \frac{x^2+3}{2x-1}$; $x \in \{1, 3, 5, 7\}$

R6	I can calculate a basic composite function.

1. Given $f(x) = x + 1$, $g(x) = x^2$ and $\boldsymbol{h(x) = x^2 - 2}$, find the following functions:

(a) $f(g(x))$	**(b)** $f(h(x))$	**(c)** $f(f(x))$
(d) $g(f(x))$	**(e)** $g(h(x))$	**(f)** $g(g(x))$
(g) $h(f(x))$	**(h)** $h(g(x))$	**(i)** $h(h(x))$

2. Given $f(x) = 2x - 3$, $g(x) = x^2$ and $h(x) = x^2 + 4$, find the following functions:

(a) $f(g(x))$	**(b)** $f(h(x))$	**(c)** $f(f(x))$
(d) $g(f(x))$	**(e)** $g(h(x))$	**(f)** $g(g(x))$
(g) $h(f(x))$	**(h)** $h(g(x))$	**(i)** $h(h(x))$

3. Given $f(x) = x^2$, $g(x) = 3x + 1$ and $h(x) = 4 - 2x$, find the following functions:

(a) $f(g(x))$	**(b)** $f(h(x))$	**(c)** $f(f(x))$
(d) $g(f(x))$	**(e)** $g(h(x))$	**(f)** $g(g(x))$
(g) $h(f(x))$	**(h)** $h(g(x))$	**(i)** $h(h(x))$

4. Given $f(x) = x - 2$, $g(x) = \frac{2}{x^2}$ and $h(x) = \frac{4}{x+1}$, find the following functions:

(a) $f(g(x))$	**(b)** $f(h(x))$	**(c)** $f(f(x))$
(d) $g(f(x))$	**(e)** $g(h(x))$	**(f)** $g(g(x))$
(g) $h(f(x))$	**(h)** $h(g(x))$	**(i)** $h(h(x))$

5. Given $f(x) = x - 2$, $g(x) = sin\, x$ and $h(x) = \log_a x$, find the following functions:

(a) $f(g(x))$ **(b)** $f(h(x))$ **(c)** $f(f(x))$

(d) $g(f(x))$ **(e)** $g(g(x))$ **(f)** $h(f(x))$

6. Given $f(x) = 2x$, $g(x) = \cos x$ and $h(x) = e^x$, find the following functions:

(a) $f(g(x))$ **(b)** $f(h(x))$ **(c)** $f(f(x))$

(d) $g(f(x))$ **(e)** $g(g(x))$ **(f)** $h(f(x))$

7. Given $f(x) = 3x^2 + 2x - 1$, $g(x) = \sin x$ and $h(x) = \log_4 x$, find the following functions:

(a) $f(g(x))$ **(b)** $f(h(x))$ **(c)** $f(f(x))$

(d) $g(f(x))$ **(e)** $g(g(x))$ **(f)** $h(f(x))$

8. Given $f(x) = x + 2$, $g(x) = e^x$ and $h(x) = \tan x$, find the following functions:

(a) $f(g(x))$ **(b)** $f(h(x))$ **(c)** $f(f(x))$

(d) $g(f(x))$ **(e)** $g(g(x))$ **(f)** $h(f(x))$

R7 I understand that $f(g(x)) = x$ implies that $g(x)$ is the inverse of $f(x)$.

1. If $f(x) = 3x - 2$ and $g(x) = \frac{x+2}{3}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

2. If $f(x) = 2x + 5$ and $g(x) = \frac{x-5}{2}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

3. If $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

4. If $f(x) = 6x - 3$ and $g(x) = \frac{x+3}{6}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

5. If $f(x) = 5x + 1$ and $g(x) = \frac{x-1}{5}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

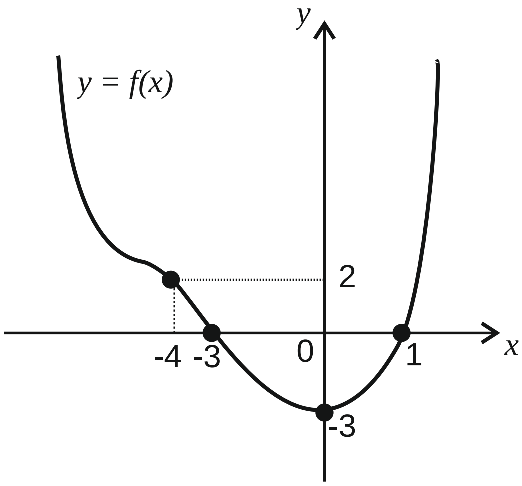
Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1	**I can identify and sketch a function after a transformation of the form $kf(x), f(x) + k, f(kx), f(x + k), -f(x), f(-x)$, or a combination of these.**

1. The diagram shows the graph of a function f.

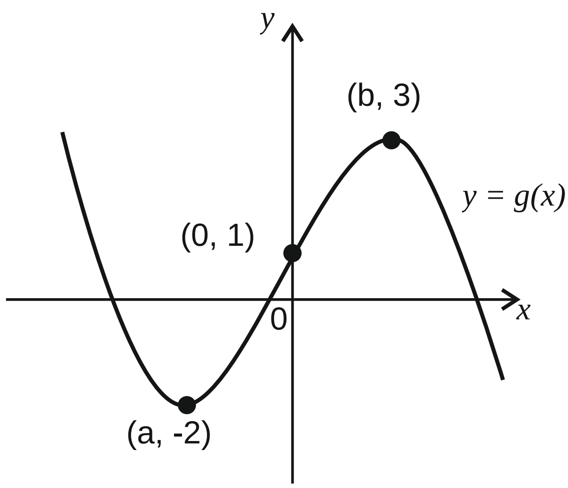
f has a minimum turning point at (0, −3) and a point of inflexion at (−4, 2).



(a) Sketch the graph $y = f(-x)$.

(b) On the same diagram, sketch the graph $y = 2f(-x)$.

2. The diagram shows the graph of $y = g(x)$.



(a) Sketch the graph of $y = -g(x)$.

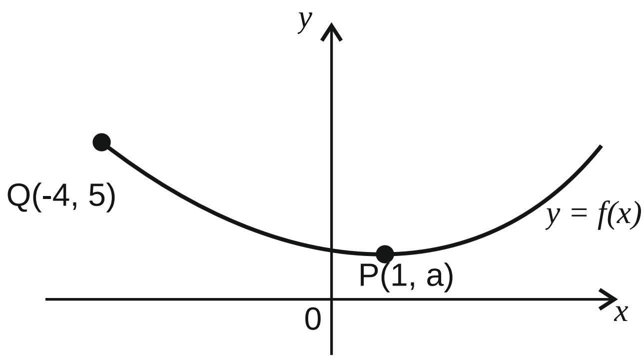
(b) On the same diagram, sketch the graph $y = 3 - g(x)$.

3. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

(a) $y = f(x-4)$.

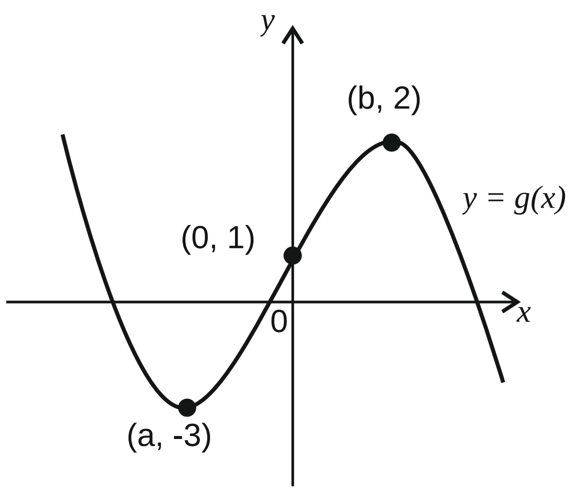
(b) $y = 2 + f(x-4)$.



4. The diagram shows the graph of $y = g(x)$.

(a) Sketch the graph of $y = -g(x)$.

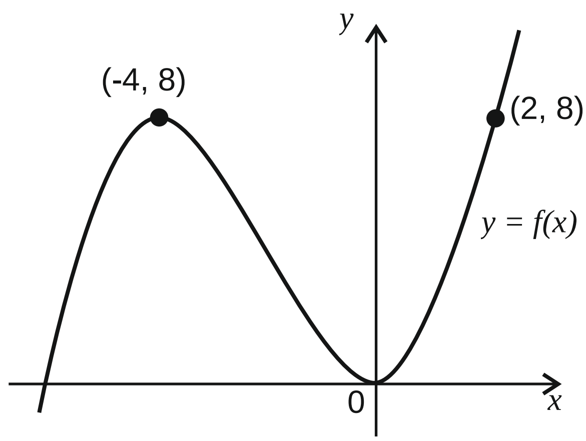
(b) On the same diagram, sketch the graph $y = 4 - g(x)$.



5. The diagram shows a sketch of the function $y = f(x)$.

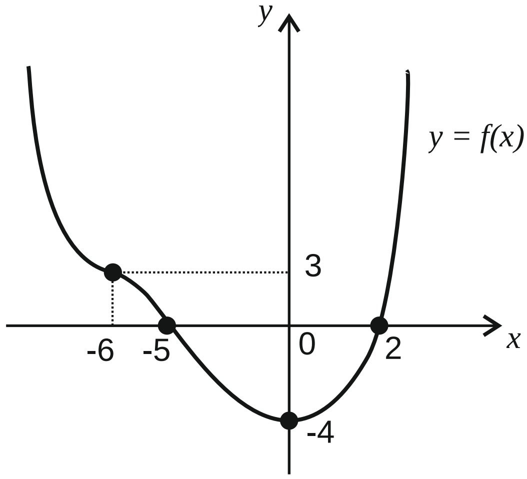
(a) Copy the diagram and on it sketch the graph of $y = f(2x)$.

(b) On a separate diagram sketch the graph of $y = 1 - f(2x)$.



6. The diagram shows the graph of a function f.

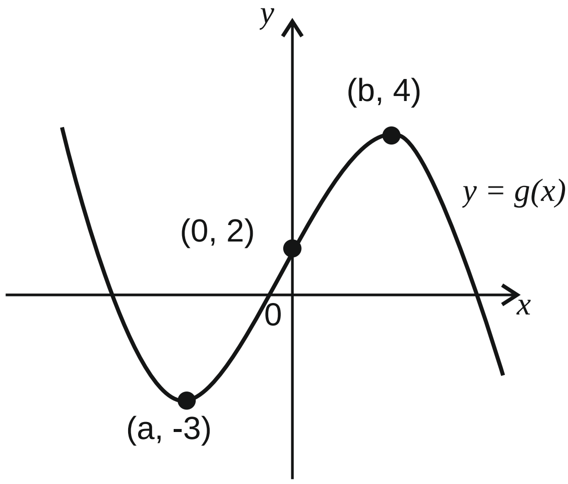
f has a minimum turning point at (0, −4) and a point of inflexion at (−6, 3).



(a) Sketch the graph $y = f(-x)$.

(b) On the same diagram, sketch the graph $y = 3f(-x)$.

7. The diagram shows the graph of $y = g(x)$.

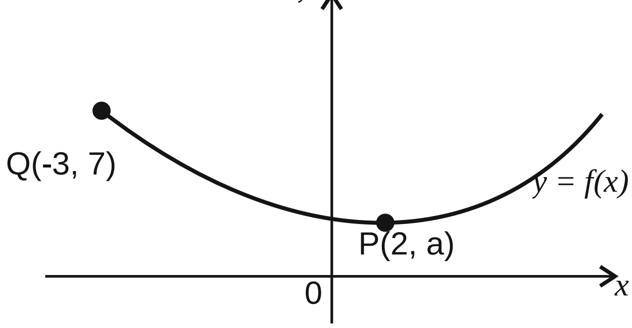


(a) Sketch the graph of $y = -g(x)$.

(b) On the same diagram, sketch the graph $y = 4 - g(x)$.

8. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:



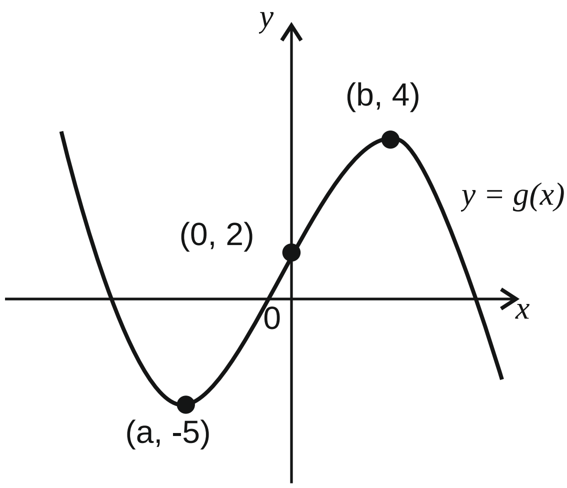
(a) $y = f(x+2)$.

(b) $y = 3 + f(x+2)$.

9. The diagram shows the graph of $y = g(x)$.

(a) Sketch the graph of $y = -g(x)$.

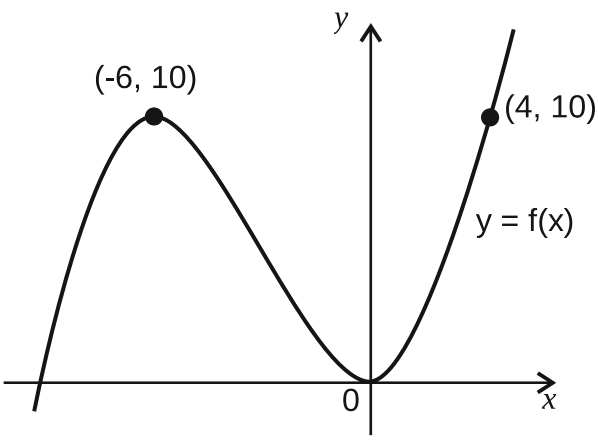
(b) On the same diagram, sketch the graph $y = 5 - g(x)$.



10. The diagram shows a sketch of the function $y = f(x)$.

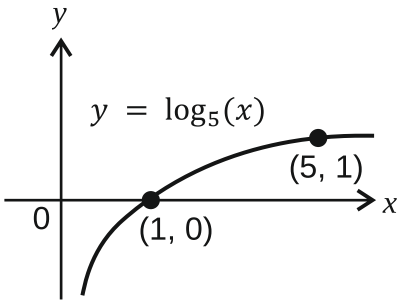
(a) Copy the diagram and on it sketch the graph of $y = f(2x)$.

(b) On a separate diagram sketch the graph of $y = 3 - f(2x)$.



NR2 I can sketch logarithmic and exponential functions and determine a suitable domain or range for a given function/composite function.

1.



The diagram shows a sketch of part of the graph of $y = \log_5 x$.

a) Make a copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 x + 1$.

Find the coordinates of the point where it crosses the *x*-axis.

b) Make a second copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$.

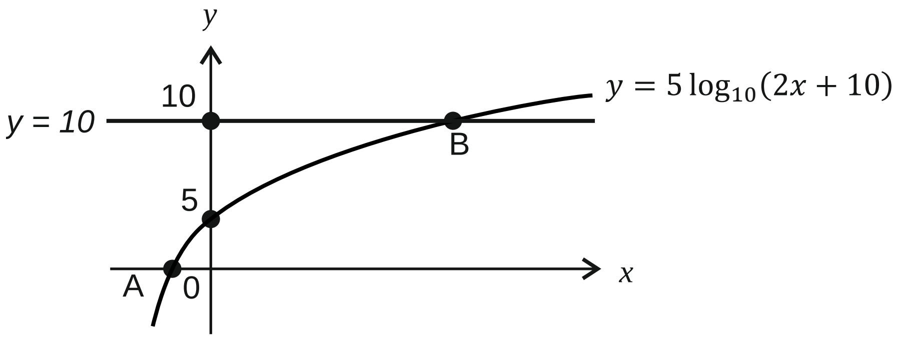
2. The functions *f* and *g*, defined on suitable domains, are given by

$f(x) = \frac{1}{x^2 - 4}$ and $g(x) = 2x + 1$.

a) Find an expression for $h(x)$ where $h(x) = g(f(x))$.
Give your answer as a single fraction.

b) State a suitable domain for *h*.

3.



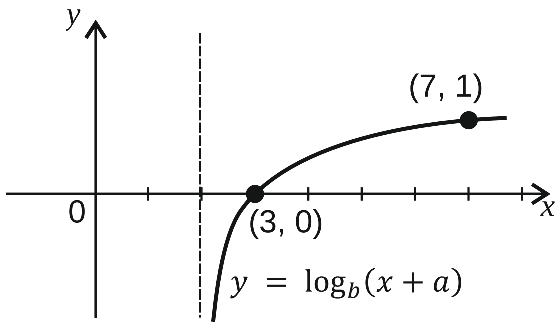
Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram (not to scale).

This graph crosses the *x*-axis at the point A and the straight line $y = 10$ at the point B.

Find algebraically the *x*-coordinates of A and B.

4. The diagram shows part of the graph of $y = \log_b(x+a)$.

Determine the values of *a* and *b*.



5. The diagram shows part of the graph of $y = 2^x$.

a) Sketch the graph of $y = 2^{-x} - 8$.

b) Find the coordinates of the points where it crosses the *x* and *y* axes.

6. a) (i) Sketch the graph of $y = a^x + 1,\ a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1},\ a > 2$

b) Prove that the graphs intersect at a point where the *x*-coordinate is $\log_a\left(\frac{1}{a-1}\right)$

7. Functions $f(x)=3x-1$ and $g(x)=x^2+7$ are defined on the set of real numbers.

a) Find $h(x)$ where $h(x)=g(f(x))$.

b) (i) Write down the coordinates of the minimum turning point y = h(x)

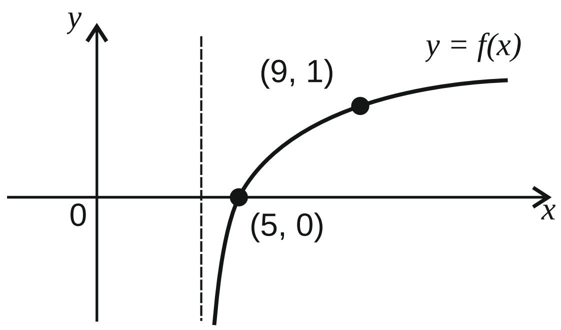
(ii) Hence state the range of the function *h*.

8. The function *f* is of the form $f(x)=\log_b(x-a)$

The graph of $y=f(x)$ is shown in the diagram.

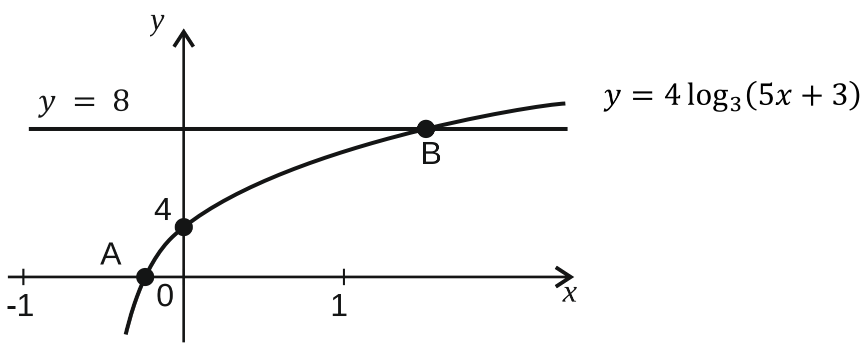
a) Write down the values of *a* and *b*.

b) State the domain of *f*.



9. Part of the graph of $y=4\log_3(5x+3)$ is shown in the diagram. This graph crosses the *x*-axis at the point A and the straight line $y=8$ at the point B.

Find the *x*-coordinate of B.



10. Sketch the following pairs of graphs on the same set of axes:

(a) $y = 2^x$ and $y = 2^x + 1$

(b) $y = a^x$ and $y = 3(a^x)$

(c) $y = 3^x$ and $y = 3^{(x+1)}$

(d) $\mathrm{y} = \log_2 x$ and $\mathrm{y} = \log_2 x(x-3)$

(e) $\mathrm{y} = \log_3 x$ and $\mathrm{y} = \log_3 x + 4$

(f) $\mathrm{y} = \log_5 x$ and $\mathrm{y} = \log_5 25$

(g) $\mathrm{y} = \log_a x$ and $\mathrm{y} = \log_a \frac{1}{x}$

(h) $\mathrm{y} = \log_4 x$ and $\mathrm{y} = 3\log_4(x+1)$

NR3 I can determine a composite function.

1. Functions f and g, determined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(a) $g(f(x))$ **(b)** $f(g(x))$ **(c)** $g(g(x))$

2. Two functions f and g, are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

3. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$, are defined on a set of real numbers..

(a) Find h(x) where $h(x) = g(f(x))$.

(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$

(ii) Hence state the range of the function h.

4. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(a) Find an expression for $h(x)$ where $h(x) = f(g(x)$.

(b) Write down any restriction on the domain of h.

5. $f(x) = 3 - x$ and $(x) = \frac{3}{x}$, $x \neq 0$

(a) Find $p(x)$ where $p(x) = f(g(x))$.

(b) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in it's simplest form.

6. Functions f and g, determined on suitable domains, are given by $f(x) = x^2 - 1$ and $g(x) = 2 - 3x$.

Find:

(a) $g(f(x))$ **(b)** $f(g(x))$ **(c)** $g(g(x))$

7. Two functions f and g, are defined by $f(x) = 2x + 1$ and $g(x) = 2x - 1$, where x is a real number.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

8. Functions $f(x) = x - 3$ and $g(x) = x^2 + 2$, are defined on a set of real numbers.

(a) Find $h(x)$ where $h(x) = g(f(x))$.

(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$

(ii) Hence state the range of the function h.

9. Functions $f(x) = \frac{1}{x+2}$ and $g(x) = 3x - 1$ are defined on suitable domains.

(a) Find an expression for $h(x)$ where $h(x) = f(g(x))$.

(b) Write down any restriction on the domain of h.

10. $f(x) = 4 - x$ and $g(x) = \frac{4}{x}, x \neq 0$

(a) Find $p(x)$ where $p(x) = f(g(x))$.

(b) If $q(x) = \frac{4}{4-x}$, $x \neq 4$, find $p(q(x))$ in it's simplest form.

NR4 I can determine the inverse of a linear function.

1. Given $f(x) = 3x - 4$, find an expression for $f^{-1}(x)$.

2. Given $g(x) = 5x + 2$, find an expression for $g^{-1}(x)$.

3. Given $h(x) = 2x - 6$, find an expression for $h^{-1}(x)$.

4. Given $f(x) = \frac{1}{2}x + 5$, find an expression for $f^{-1}(x)$.

5. Given $g(x) = \frac{1}{4}x - 3$, find an expression for $g^{-1}(x)$.

6. Given $h(x) = 7 - 3x$, find an expression for $h^{-1}(x)$.

7. Given $f(x) = 2 - 4x$, find an expression for $f^{-1}(x)$.

8. Given $g(x) = \frac{2x - 4}{5}$, find an expression $g^{-1}(x)$.

9. Given $h(x) = \frac{3x + 2}{4}$, find an expression for $h^{-1}(x)$.

10. Given $f(x) = \frac{6 - 3x}{2}$, find an expression for $f^{-1}(x)$.

NR5 I have experience of cross topic exam standard questions.

1. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$

(a) Find $g(f(x))$

(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$

(c) **(i)** Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.

(d) Hence solve $g(f(x)) + xh(x) = 0$

2. Functions f and g are defined on a suitable domain by $f(x) = sin\,(x°)$ and $g(x) = 2x$

(a) Find expressions for

(i) $f(g(x))$

(ii) $g(f(x))$

(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$

3. Functions f, g and h are defined on suitable domains by

$f(x) = x^2 - x + 10$ $\quad$ $g(x) = 5 - x$ $\quad$ and $h(x) = \log_2 x$

(a) Find expressions for h(f(x)) and h(g(x)).

(b) Hence solve $h(f(x)) - h(g(x)) = 3$.

4. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for:

(i) $f(h(x))$

(ii) $g(h(x))$

(b) **(i)** $h(f(x))$

(ii) $h(g(x))$

5. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$

(a) **(i)** Find $p(x)$ where $p(x) = f(g(x))$

(ii) Find $q(x)$ where $q(x) = g(f(x))$

(b) Solve $p'(x) = q'(x)$

6. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$

(a) Find expressions for

(i) $f(g(x))$

(ii) $g(f(x))$

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.

7. Functions $a(x) = \sin x$, $b(x) = \cos x$ and $c(x) = x - \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for;

(i) $a(c(x))$;

(ii) $b(c(x))$.

(b) **(i)** Show that $a(c(x)) = \frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $b(c(x))$ and hence solve the equation $a(c(x)) + b(c(x)) = 1$ for $0 \leq x \leq 2\pi$.

8. Functions f and g are defined on suitable domains by $f(x) = \sin x°$ and $g(x) = 2x$.

(a) Find expressions for;

(i) $f(g(x))$;

(ii) $g(f(x))$.

(b) Solve $3f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$.

Vectors
Higher Mathematics Supplementary Resources

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have revised National 5 vectors and 3D coordinate.

1. If vector $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find the resultant vector of:

(a) $\boldsymbol{a} + \boldsymbol{b}$ **(b)** $\boldsymbol{a} - \boldsymbol{b}$ **(c)** $3\boldsymbol{a} + \boldsymbol{b}$

(d) $\boldsymbol{a} - 2\boldsymbol{b}$ **(e)** $5\boldsymbol{a} - 3\boldsymbol{b}$ **(f)** $2\boldsymbol{a} + 4\boldsymbol{b}$

2. If vector $\boldsymbol{a} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$, find the resultant vector of

(a) $\boldsymbol{a} + \boldsymbol{b}$ **(b)** $\boldsymbol{a} - \boldsymbol{b}$ **(c)** $2\boldsymbol{a} + 3\boldsymbol{b}$

(d) $5\boldsymbol{a} - \boldsymbol{b}$ **(e)** $3\boldsymbol{a} - 2\boldsymbol{b}$ **(f)** $\boldsymbol{a} + 4\boldsymbol{b}$

3. If vector $\boldsymbol{p} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ and vector $\boldsymbol{q} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$, find the resultant vector of

(a) $\boldsymbol{p} + \boldsymbol{q}$ **(b)** $\boldsymbol{p} - \boldsymbol{q}$ **(c)** $\boldsymbol{p} + 2\boldsymbol{q}$

(d) $2\boldsymbol{p} - \boldsymbol{q}$ **(e)** $3\boldsymbol{p} - 5\boldsymbol{q}$ **(f)** $4\boldsymbol{p} + 3\boldsymbol{q}$

4. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$, find:

(a) $|\boldsymbol{p}|$ **(b)** $|\boldsymbol{q}|$ **(c)** $|\boldsymbol{p} + \boldsymbol{q}|$

(d) $|\boldsymbol{p} - \boldsymbol{q}|$ **(e)** $|3\boldsymbol{p} - \boldsymbol{q}|$ **(f)** $|2\boldsymbol{p} + 3\boldsymbol{q}|$

5. Three vectors are defined as $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$ and $\overrightarrow{EF} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$, find:

(a) $|\overrightarrow{AB}|$ **(b)** $|\overrightarrow{CD}|$ **(c)** $|\overrightarrow{EF}|$

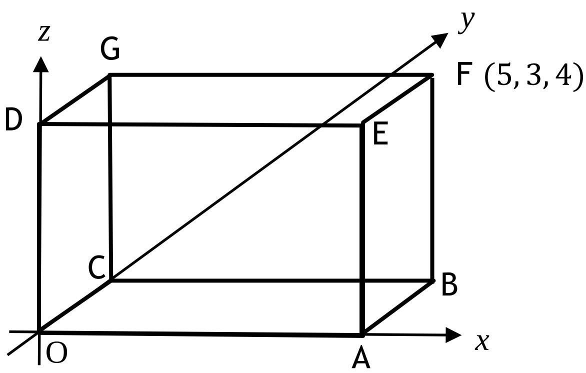
6. Three points A, B and C have the coordinates $(2, 5, 3)$, $(-1, 3, 0)$ and $(1, 4, 2)$ respectively. Find the vectors

(a) $\overrightarrow{OA}$ **(b)** $\overrightarrow{OB}$ **(c)** $\overrightarrow{OC}$

(d) $\overrightarrow{AB}$ **(e)** $\overrightarrow{BC}$ **(f)** $\overrightarrow{AC}$

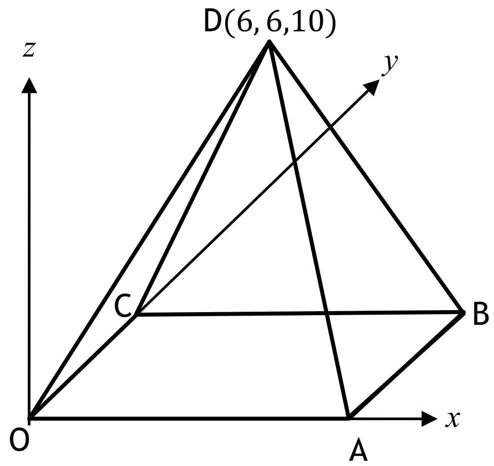
7. The diagram shows the cuboid OABCDEFG. O is the origin and OA, OC and OD are aligned with the x, y and z axes respectively. The point F has coordinates $(5, 3, 4)$.

List the coordinates of the other six vertices.



8. The diagram shows the square based pyramid DOABC. O is the origin with OA and OC aligned with the x and y axes respectively. The point D has coordinates $(6, 6, 10)$.

Write down the coordinates of the points A, B and C.



R2 I can express and manipulate vectors in the form $a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$.

1. **Write the following vectors, given in unit vector form, in component form.**

(a) $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ (b) $\boldsymbol{b} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ (c) $\boldsymbol{c} = 4\boldsymbol{i} + 2\boldsymbol{j}$

(d) $\boldsymbol{d} = 3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ (e) $\boldsymbol{e} = \boldsymbol{i} - 6\boldsymbol{j} - 4\boldsymbol{k}$ (f) $\boldsymbol{f} = -\boldsymbol{i} + 3\boldsymbol{k}$

2. **Write the following vectors, given in component form, in unit vector form.**

(a) $\boldsymbol{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (b) $\boldsymbol{q} = \begin{pmatrix} 6 \\ -2 \\ 7 \end{pmatrix}$ (c) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$

(d) $\boldsymbol{s} = \begin{pmatrix} 9 \\ 0 \\ -3 \end{pmatrix}$ (e) $\boldsymbol{t} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (f) $\boldsymbol{u} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

3. Two vectors are defined, in unit vector form, as $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$.

(a) Express $\boldsymbol{a} + \boldsymbol{b}$ **in unit vector form.**

(b) Express $2\boldsymbol{a} - \boldsymbol{b}$ **in unit vector form.**

(c) Find $|\boldsymbol{a} + \boldsymbol{b}|$.

(d) Find $|2\boldsymbol{a} - \boldsymbol{b}|$.

4. Two vectors are defined, in unit vector form, as $\boldsymbol{p} = 3\boldsymbol{i} - \boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$.

(a) Express $\boldsymbol{p} + 2\boldsymbol{q}$ **in unit vector form.**

(b) Express $3\boldsymbol{p} - 4\boldsymbol{q}$ **in unit vector form.**

(c) Find $|\boldsymbol{p} + 2\boldsymbol{q}\ |$.

(d) Find $|3\boldsymbol{p} - 4\boldsymbol{q}|$.

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1	**I can determine whether or not coordinates are collinear, using the appropriate language, and can apply my knowledge of vectors to divide lines in a given ratio.**

1. The point Q divides the line joining P$(-1, -1, 3)$ and R$(5, -1, -3)$ in the ratio 5:1. Find the coordinates of Q.

2. John is producing a 3D design on his computer.

Relative to suitable axes 3 points in his design have coordinates P$(-3, 4, 7)$, Q$(-1, 8, 3)$ and R$(0, 10, 1)$.

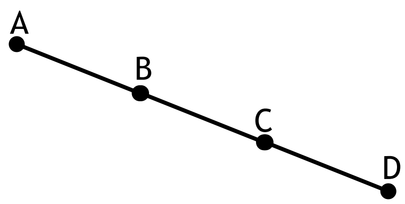
(a) Show that P, Q and R are collinear.

(b) Find the coordinates of S such that $\overrightarrow{PS} = 4\overrightarrow{PQ}$.

3. A and B are the points $(0, -2, 3)$ and $(3, 0, 2)$ respectively.

B and C are the points of trisection of AD, that is AB = BC = CD.

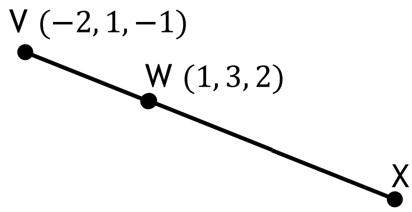
Find the coordinates of D.



4. The points V, W and X are shown on the line opposite.

V, W and X are collinear points such that WX = 2VW.

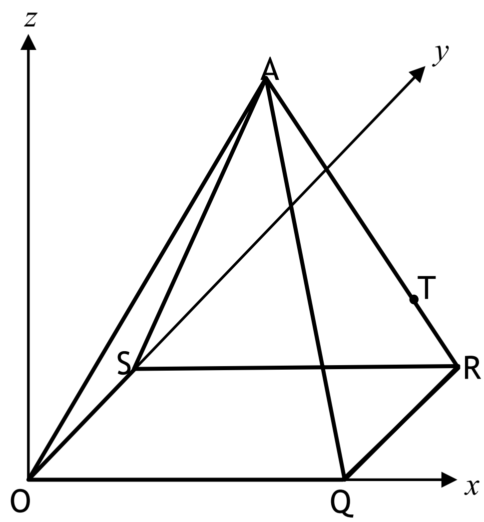
Find the coordinates of X.



5. AOQRS is a pyramid. Q is the point $(16, 0, 0)$, R is $(16, 8, 0)$ and A is $(8, 4, 12)$. T divides RA in the ratio 1:3.

(a) Find the coordinates of the point T.

(b) Express $\overrightarrow{QT}$ in component form.



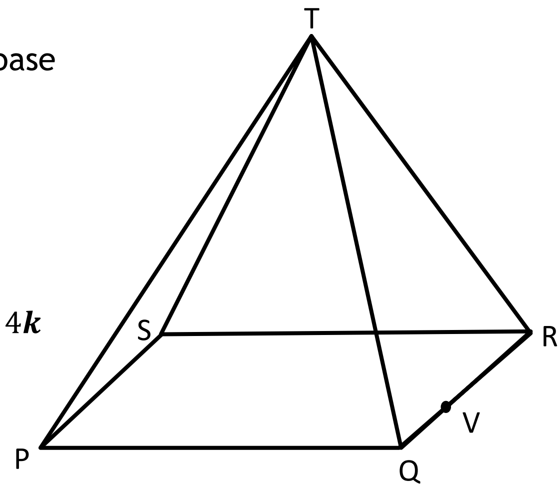
6. PQRST is a pyramid with a rectangular base PQRS.

V divides QR in the ratio 1:3 and

$\overrightarrow{TP} = -7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$,

$\overrightarrow{PQ} = 6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$ and $\overrightarrow{PS} = 8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$

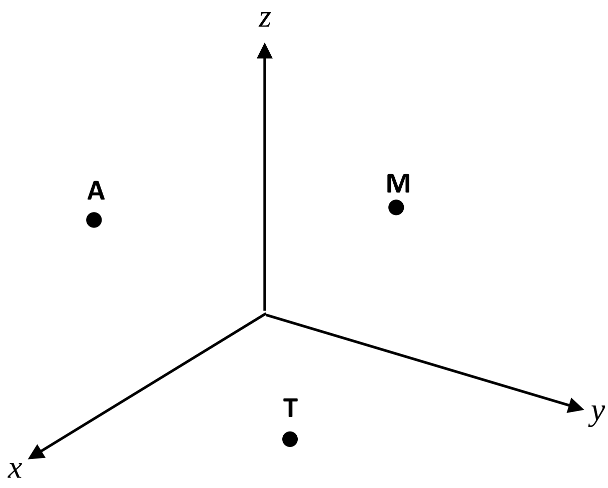
Find $\overrightarrow{TV}$ in component form.



NR2 **I can apply knowledge of vectors to find an angle in three dimensions.**

1. A surveyor is checking a room for movement in the walls due to subsidence. She sets up 3 points. Two of the points, A and M, are on two different walls which meet perpendicularly along the z axis, relative to the axes shown. The other point T is on the floor.

The three points have coordinates $(6, 0, 7)$, $(0, 5, 6)$ and $(4, 5, 0)$.



(a) Match the three points to the correct coordinates.

(b) Write $\overrightarrow{TA}$ and $\overrightarrow{TM}$ in component form.

(c) Find the size of angle ATM.

2. V, W and X have coordinates $(1, 3, -1)$, $(2, 0, 1)$ and $(-3, 1, 2)$ respectively.

(a) Find $\overrightarrow{VW}$ and $\overrightarrow{VX}$ in component form.

(b) Hence find the size of angle WVX.

3. Three planes, Tango (T), Delta (D) and Bravo (B) are being tracked by radar. Relative to a suitable origin, the positions of the three planes are T$(23, 0, 8)$, D$(-12, 0, 9)$ and B$(28, -15, 7)$

(a) Express the vectors $\overrightarrow{BT}$ and $\overrightarrow{BD}$ in component form.

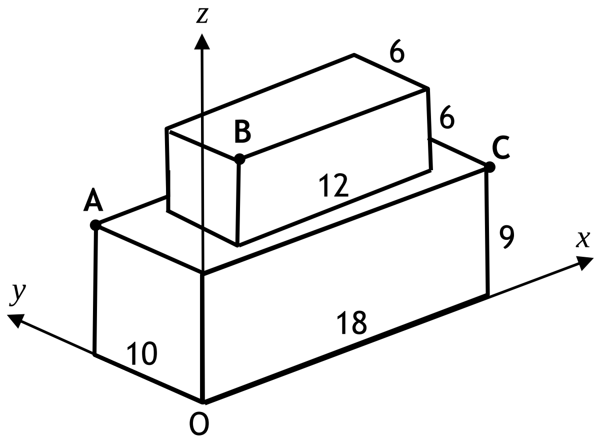
(b) Find the size of angle TBD.

4. A cuboid measuring 12cm by 6cm by 6cm is placed centrally on top of another cuboid measuring 18cm by 10cm by 9cm.

Coordinate axes are taken as shown.

(a) The point A has coordinates $(0, 10, 9)$ and the point C has coordinates $(18, 0, 9)$. Write down the coordinates of B.

(b) Find the size of angle ABC.



5. A square-based pyramid, OABCD, has a height of 10 units and the square base has a length of 8 units.

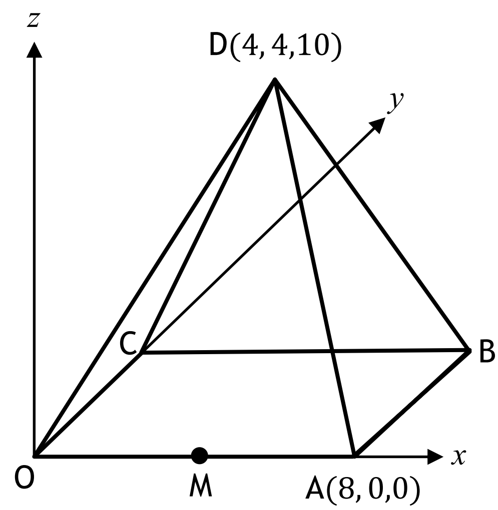
The coordinates of two points, A and D are shown on the diagram.

(a) Write down the coordinates of the point B.

(b) Determine the components of the vectors $\overrightarrow{DA}$ and $\overrightarrow{DB}$.

(c) Find the size of angle ADB.

M is the midpoint of OA.



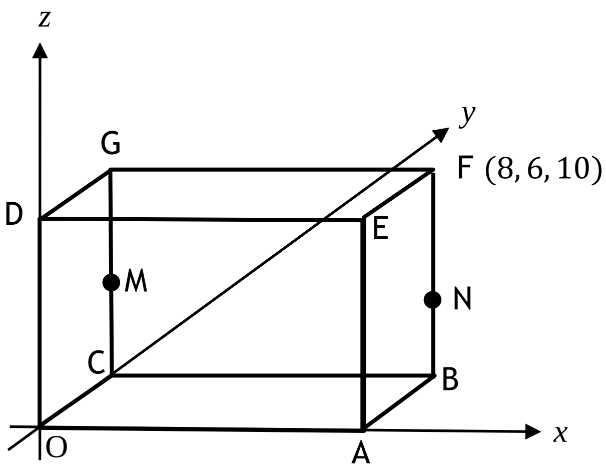
(d) Write down the coordinates of C and M.

(e) Determine the components of the vectors $\overrightarrow{DC}$ and $\overrightarrow{DM}$.

(f) Find the size of angle CDM.

6. The diagram shows a cuboid OABCDEFG with the lines OA, OC and OD lying on the axes.

The point F has coordinates $(8, 6, 10)$, M is the midpoint of CG and N divides BF in the ratio 2:3.



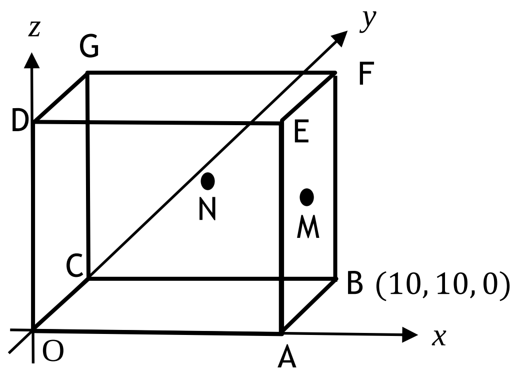
(a) State the coordinates of A, M and N.

(b) Determine the components of the vectors $\overrightarrow{MA}$ and $\overrightarrow{MN}$.

(c) Find the size of angle AMN.

7. The diagram shows a cube OABCDEFG. B has coordinates $(10, 10, 0)$

M is the centre of AEFB and N is the centre of face GFBC.

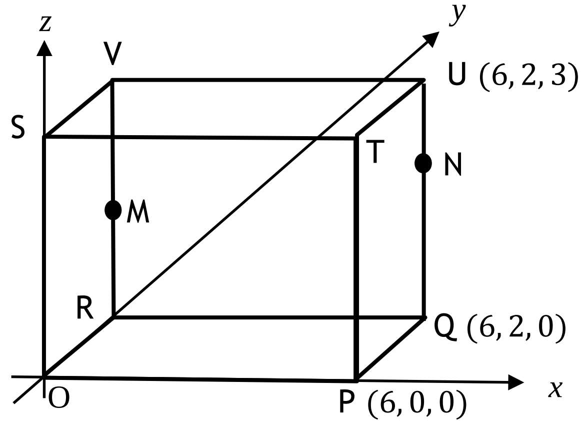


(a) Write down the coordinates of G.

(b) Find $\boldsymbol{m}$ and $\boldsymbol{n}$, the position vectors of M and N.

(c) Find the size of angle MON.

8. In the diagram OPQRSTUV is a cuboid. M is the midpoint of VR and N is the point on UQ such that $\mathrm{UN} = \frac{1}{3}\mathrm{UQ}$.



(a) State the coordinates of T, M and N.

(b) Determine the components of the vectors $\overrightarrow{TM}$ and $\overrightarrow{TN}$.

(c) Find the size of angle MTN.

NR3 I know the properties of the scalar product and their uses.

1. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are defined by $\boldsymbol{p} = -3\boldsymbol{i} - 12\boldsymbol{k}$ and $\boldsymbol{q} = 8\boldsymbol{i} + 7\boldsymbol{j} - 2\boldsymbol{k}$.

   Determine whether or not $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular to each other.

2. For what value of p are the vectors $\boldsymbol{a} = \begin{pmatrix} p \\ -2 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 14 \\ 2p \end{pmatrix}$ perpendicular?

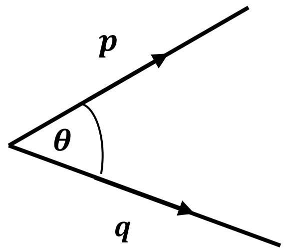
3. A and B have coordinates $(9, -7, -14)$, $(0, -1, -3)$ respectively.

   C has coordinates $(k, 0, -1)$.

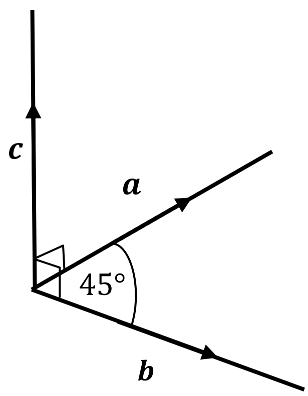
   Given that AB is perpendicular to CB, find the value of k.

4. The diagram shows vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

   If $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.(\boldsymbol{p} + \boldsymbol{q}) = 15$, find the size of the acute angle θ between $\boldsymbol{p}$ and $\boldsymbol{q}$.



5.



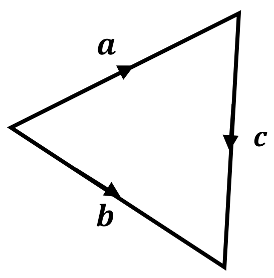
The diagram shows vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.

$|\boldsymbol{a}| = 3$, $|\boldsymbol{b}| = \sqrt{2}$ and the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is $45°$.

$\boldsymbol{c}$ is perpendicular to $\boldsymbol{a}$ and to $\boldsymbol{b}$.

Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.

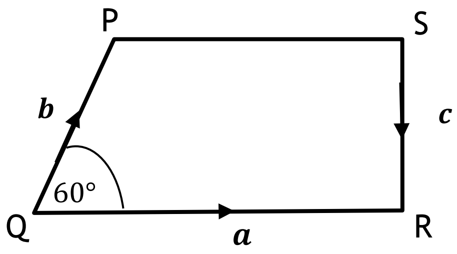
6. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ form an equilateral triangle of length 3 units.



(a) Find the scalar product $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$.

(b) What does this tells us about the vectors $\boldsymbol{a}$ and $\boldsymbol{b}+\boldsymbol{c}$.

7. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are shown on the diagram. Angle PQR = $60°$.



It is also given that $|\boldsymbol{a}| = \mathbf{3}$ **and** $|\boldsymbol{b}| = \mathbf{2}$.

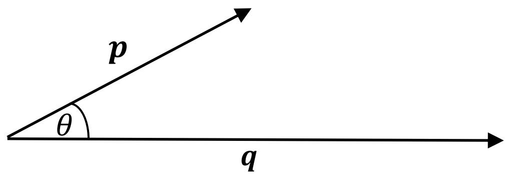
(a) **Evaluate** $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ **and** $\boldsymbol{c}.(\boldsymbol{a}-\boldsymbol{b})$.

(b) **Find** $|\boldsymbol{b}+\boldsymbol{c}|$ **and** $|\boldsymbol{a}-\boldsymbol{b}|$.

NR4 I have experience of cross topic exam standard questions.

Vectors and Polynomials

1. $\boldsymbol{p}$ and $\boldsymbol{q}$ are vectors given by $\boldsymbol{p} = \begin{pmatrix} k^2 \\ 3 \\ k+1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} k \\ k^2 \\ -2 \end{pmatrix}$, where $k > 0$.



(a) If $\boldsymbol{p} \cdot \boldsymbol{q} = 1 - k$, show that $k^3 + 3k^2 - k - 3 = 0$.

(b) Show that $(k+3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise fully.

(c) Deduce the only possible value of k.

Vectors and Quadratics

1. P is the point $(1, -3, 0)$, Q$(1, -1, 2)$ and R$(k, -2, 0)$

(a) Express $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form.

(b) Show that $\cos P\hat{Q}R = \frac{3}{\sqrt{2(k^2-2k+6)}}$

(c) If angle $PQR = 30°$, find the possible values of k.