Higher Mathematics

Supplementary Material

Relationships and Calculus

POLYNOMIALS

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can factorise a polynomial expression using the factor theorem.

1. Show that $x = -4$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

Hence factorise $x^3 + 8x^2 + 11x - 20$ fully.

2. (a) Show that $(x + 2)$ is a factor of $f(x) = 2x^3 + 3x^2 - 5x - 6$.

(b) Hence factorise $f(x)$ fully.

3. (a) Show that $(x + 1)$ is a factor of $f(x) = 2x^3 - 3x^2 - 3x + 2$.

(b) Hence factorise $f(x)$ fully.

4. Show that $(x - 2)$ is a factor of $f(x) = x^3 - 5x^2 + 2x + 8$.

(a) Factorise $x^3 - 5x^2 + 2x + 8$ fully

(b) Solve $x^3 + 2x = 5x^2 - 8$

5. Factorise fully

a) $x^3 - 7x + 6$
b) $2x^3 + 3x^2 - 2x - 3$
c) $2x^3 - x^2 - 13x - 6$
d) $3x^3 + 8x^2 - 5x - 6$
e) $2x^4 + 6x^3 + 6x^2 + 2x$
f) $x^5 + x^4 - x - 1$

NR2 I can evaluate an unknown coefficient of a polynomial by applying the remainder and/or the factor theorem.

1. $f(x) = 2x^3 + ax^2 + bx + 4$.
   Given that $(x - 2)$ is a factor of $f(x)$, and the remainder when $f(x)$ is divided by $(x - 5)$ is 54, find the values of *a* and *b*.

2. Find p if $(x - 4)$ is a factor of $x^3 - 9x^2 + px - 28$.

3. Given that (x + 1) is a factor of $2x^3 + 3x^2 + px - 6$
   (a) Find the value of p
   (b) Hence or otherwise, solve $2x^3 + 3x^2 + px - 6 = 0$

4. Find the value of k if (x+5) is a factor of $3x^4 + 15x^3 - kx^2 - 9x + 5$

5. Given that (x-1) is a factor of $x^3 + x^2 - (t + 1)x - 4$, find the value of 't' and hence factorise fully.

6. Given that x = 3 is a root of the equation $x^4 - 3x^3 + px - 5$, find p.

7. When $x^4 - 3x^3 + px - 5$ is divided by (x+3) the remainder is 16.

   Find the value of p.

NR3 I can solve a polynomial equation to determine where a curve cuts the x-axis.

1. A function is defined on the set of real numbers by $f(x) = (x+3)(x^2+4)$.

Find where the graph of $y = f(x)$ cuts:

(a) the x-axis;

(b) the y-axis. **(Non-calculator)**

2. A function is defined by the formula $g(x) = 2x^3 - 7x^2 - 17x + 10$ where x is a real number.

(a) Show that $(x-5)$ is a factor of $g(x)$, and hence factorise $g(x)$ fully.

(b) Find the coordinates of the points where the curve with

equation $y = g(x)$ crosses the x and y-axes. **(Non-calculator)**

3. A function is defined by the formula $f(x) = 5x - x^3$.

Find the exact values where the graph of $y = f(x)$ meets the x and y-axes.

(Non-calculator)

4. $h(x) = x^3 - 5x^2 + 3x + 9$

(a) (i) Show that $(x+1)$ is a factor of $h(x)$.

(ii) Hence or otherwise factorise $h(x)$ fully.

(b) One of the turning points of the graph of $y = h(x)$ lies on the x-axis.

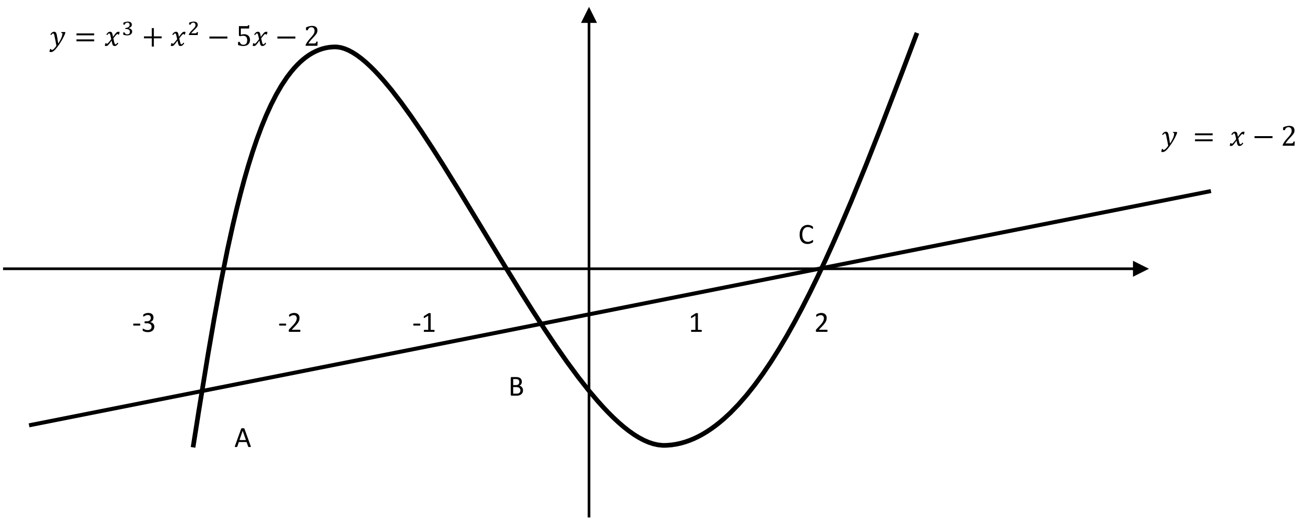
Write down the coordinates of this turning point. **(Non-calculator)**

5. Find where the graph of $y = x^4 + 6x^3 - 12x^2 - 88x - 96$ meets the

x and y-axes. **(Non-calculator)**

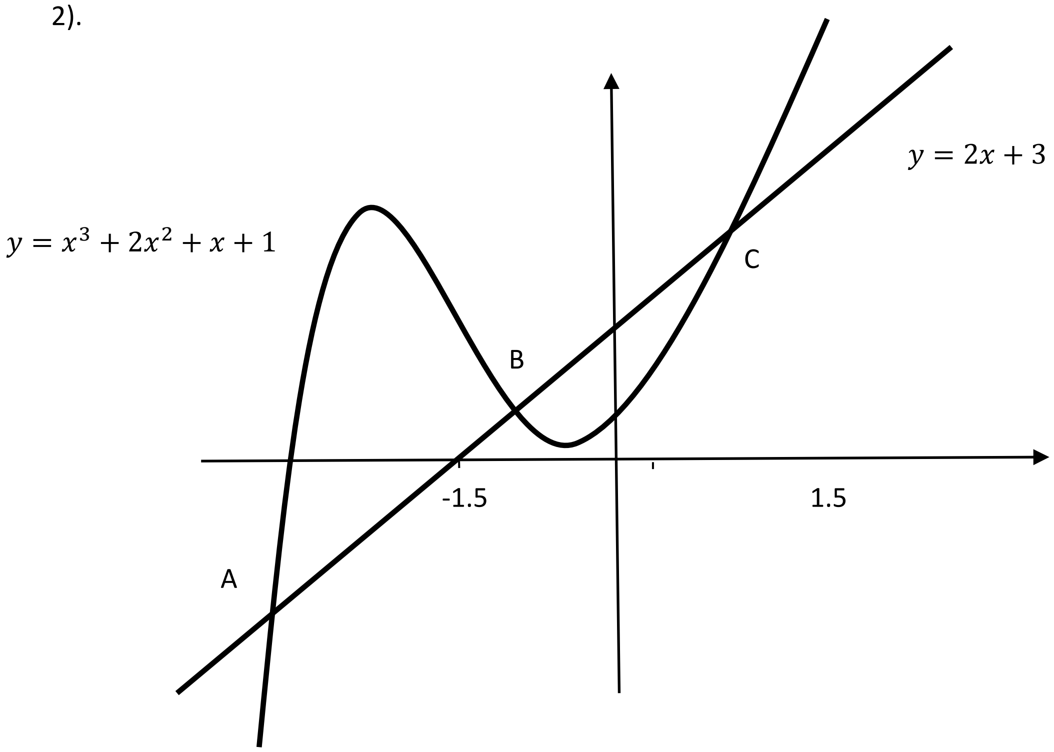
NR4 I can find points of intersection by solving polynomial equations.

1).



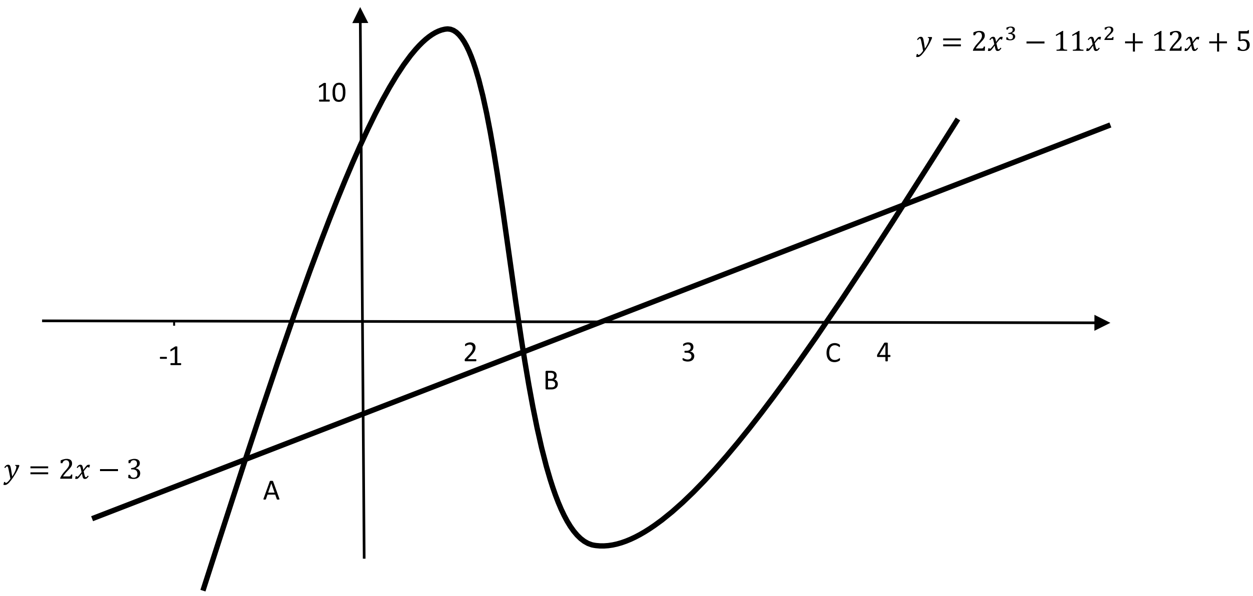
Find the coordinates of the points of intersection namely A, B and C where the line $y = x - 2$ meets the graph $y = x^3 + x^2 - 5x - 2$. (**NB the diagram is not drawn to scale**)

2).



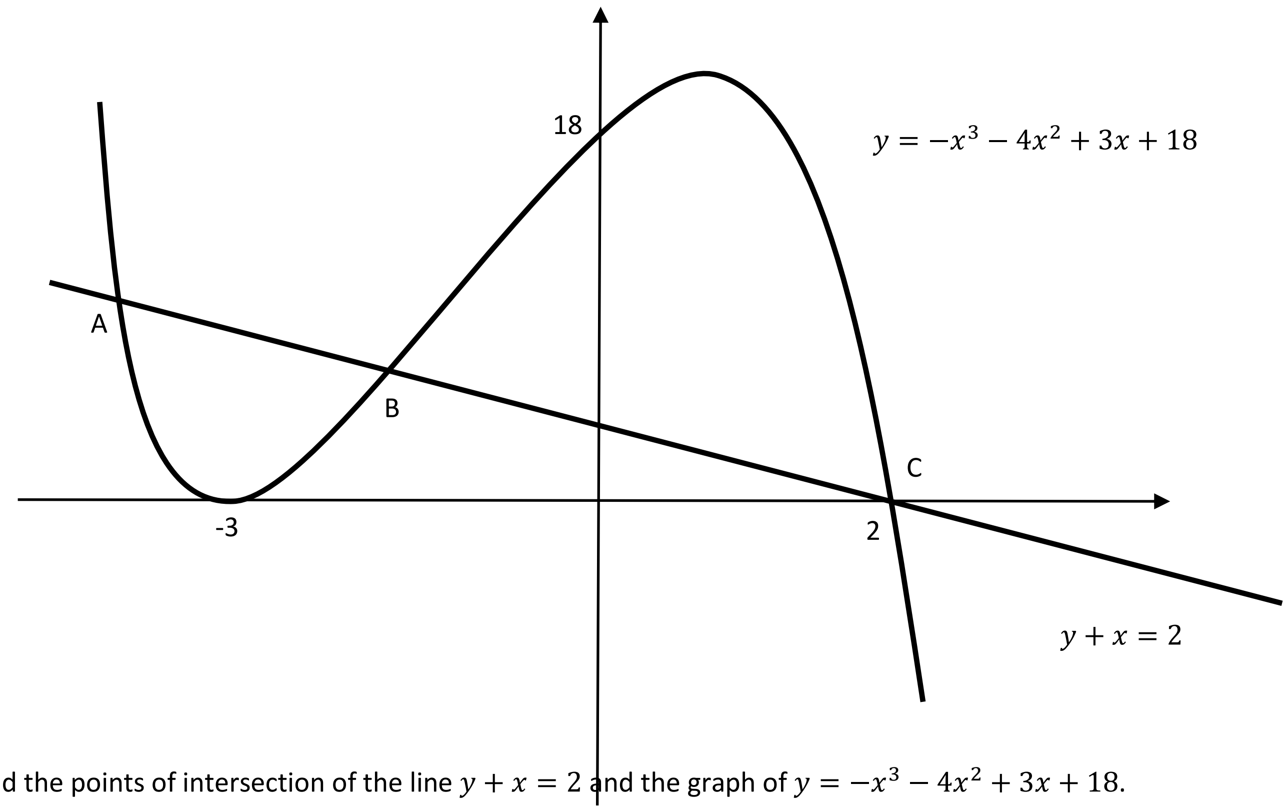
Find the coordinates A, B and C of the points of intersection between the line $y = 2x + 3$ and the curve $y = x^3 + 2x^2 + x + 1$.

3) (NB diagram not to scale)



a) Fully factorise the polynomial $2x^3 - 11x^2 + 10x + 8$
b) Hence or otherwise find the coordinates of the points of intersection of the line $y = 2x - 3$ and the graph $y = 2x^3 - 11x^2 + 12x + 5$

4)



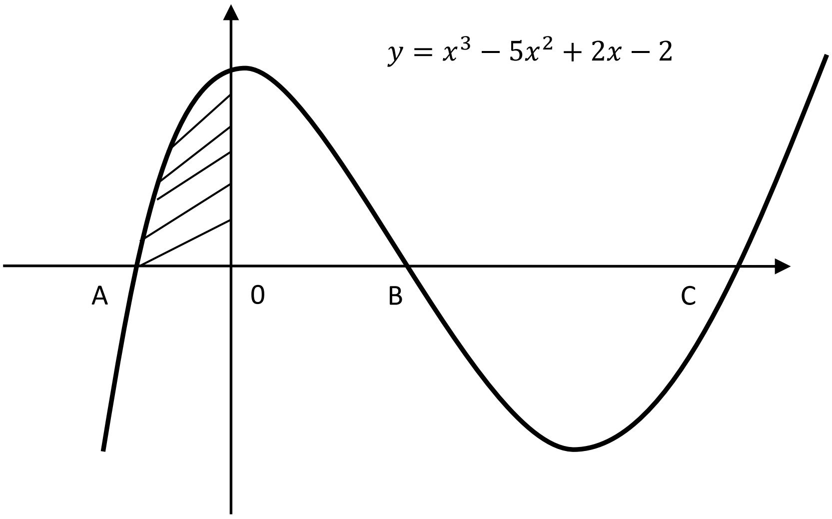
Find the points of intersection of the line $y + x = 2$ and the graph of $y = -x^3 - 4x^2 + 3x + 18$.

NR5 I have experience of cross topic exam standard questions.

1. Given that $f'(x) = 6x^2 + 2x - 128$ and $(x - 8)$ is a factor of $f(x)$, find
   a) Formula for $f(x)$
   b) Hence factorise $f(x)$ fully
   c) Solve $f(x) = 0$.

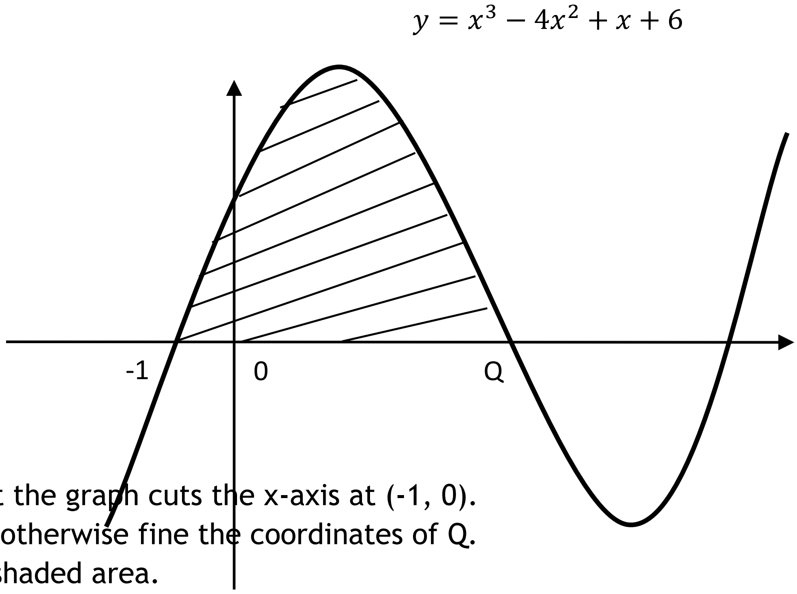
2. (a) (i) Show that $(x - 2)$ is a factor of $x^3 - 5x^2 + 2x + 8$.
   (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.
   (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

   b)The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at A, B and C.
Determine the shaded area.

3. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$



a) Show that the graph cuts the x-axis at (-1, 0).
b) Hence or otherwise fine the coordinates of Q.
c) Find the shaded area.

4. Functions $f, g\ and\ h$ are defined on the set of real numbers by

- $f(x) = x^3 + 1$
- $g(x) = 2x + 1$
- $h(x) = 17 - 11x$

a) Find $g(f(x))$

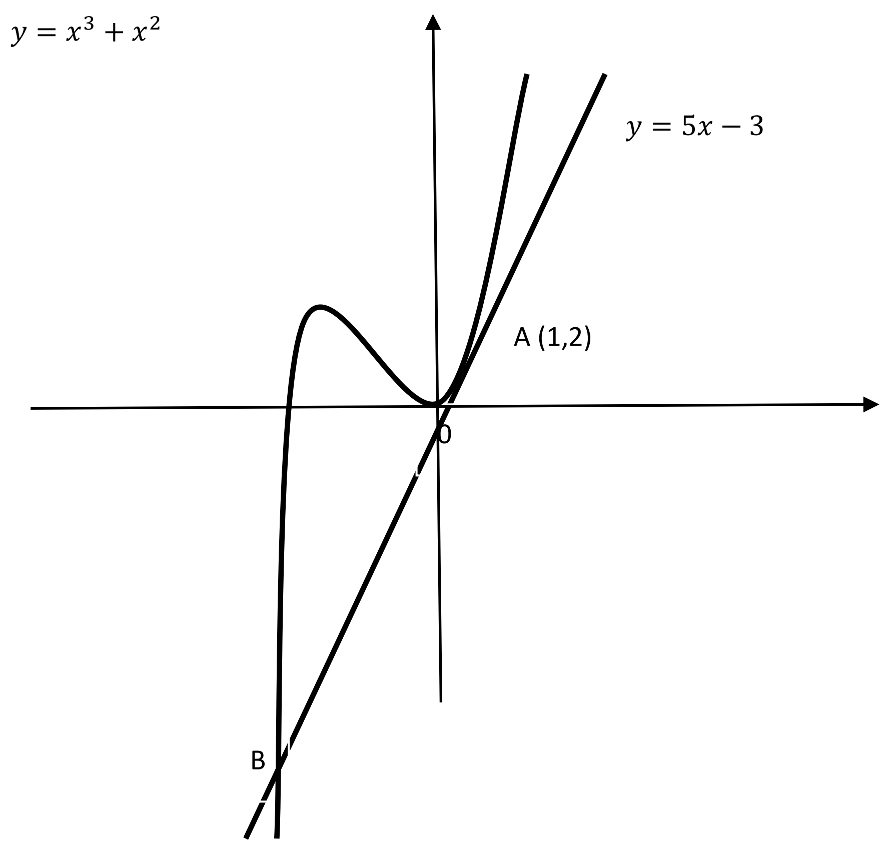
b) Show that $g(f(x)) + xh(x) - 9 = 2x^3 - 11x + 17x - 6$

c) (i) Show that $(x - 2)$ is a factor of $2x^3 - 11x + 17x - 6$

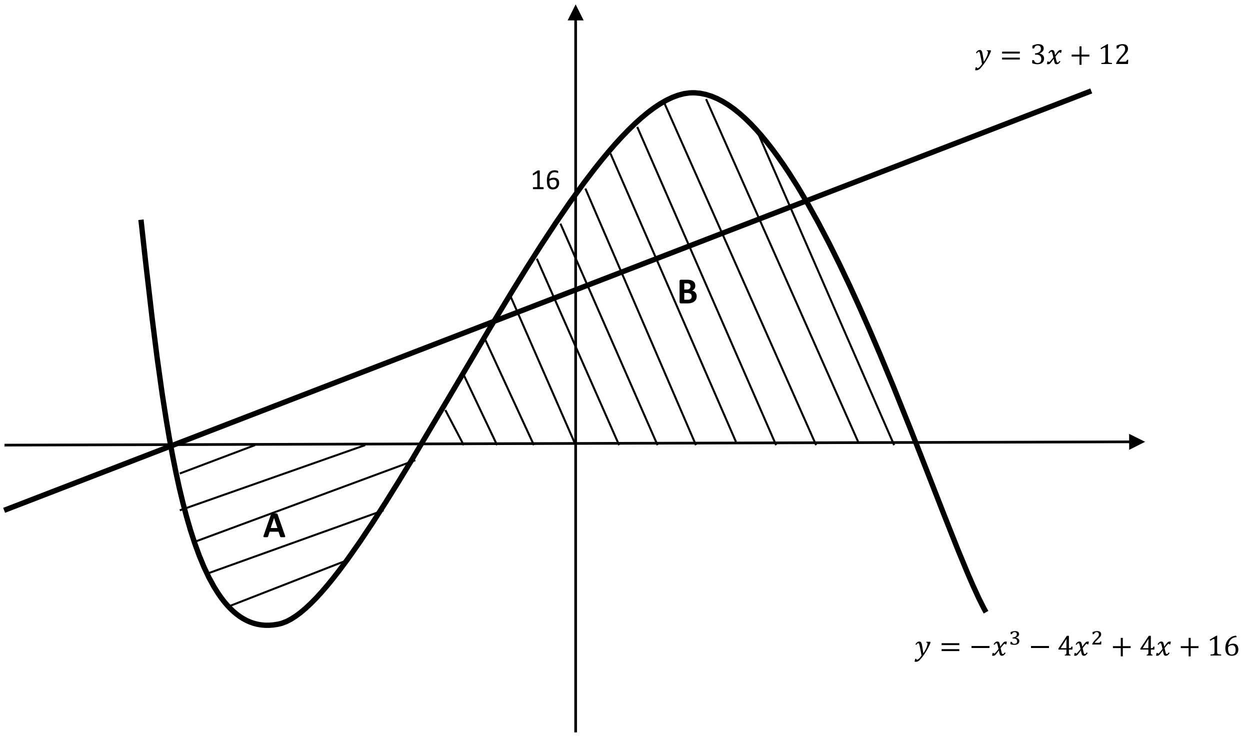
(ii) Factorise $2x^3 - 11x + 17x - 6$

d) Hence or otherwise solve $g(f(x)) + xh(x) = 9$

5. The line with equation y = 5x – 3 is a tangent to the curve with equation $y = x^3 + x^2$ at A (1,2), as shown on the diagram. Show that the point B is (-3,-18)



6.



a) Find where the graph $y = -x^3 - 4x^2 + 4x + 16$ crosses the x axis.
b) Calculate the points of intersection of the line $y = 3x + 12$ and the graph.
c) Find the shaded area A
d) Find the shaded area B

QUADRATIC THEORY

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I have had experience of factorising. (common factor, difference of two squares and trinomial quadratics).

1. Factorise fully

(a) $98 - 8x^2$ (b) $5s^2 - 5t^2$ (c) $98 - 2x^2$

(d) $75x^2 - 243$ (e) $72 - 18x^2$ (f) $12x - 3x^3$

(g) $81 - x^4$ (h) $27w - 12w^3$ (i) $64a^4 - 4$

(j) $50x^3 - 2x$ (k) $5r^3 - 20r$ (l) $32p^5 - 2p$

2. Factorise fully

(a) $2x^2 - 7x + 3$ (b) $2x^2 + 11x + 12$ (c) $3x^2 + 10x + 8$

(d) $x^2 + x - 6$ (e) $6x^2 + 7x + 2$ (f) $x^2 - 3x + 2$

(g) $5x^2 + 4x - 1$ (h) $7x^2 + 16x + 4$ (i) $2x^2 + 7x - 15$

(j) $x^2 - 2x - 15$ (k) $4x^2 + 13x + 3$ (l) $12x^2 - 4x - 1$

(m) $8x^2 + 2x - 3$ (n) $8x^2 + 6x - 9$ (o) $9x^2 + 15x + 4$

3. Factorise fully

(a) $6 - x - x^2$ (b) $20 + 11x - 3x^2$ (c) $3 + x - 2x^2$

(d) $15 - 7x - 2x^2$ (e) $4 - 7x - 2x^2$ (f) $15 - 2x - x^2$

4. Factorise fully

(a) $3x^2 + 6x - 24$ (b) $15x^2y + 5x$ (c) $2x^2 - 32$

(d) $5x^3 - 45x$ (e) $18x^2 - 6x - 12$ (f) $12x^2y + 8xy^3$

(g) $10x^2 + 25x - 15$ (h) $6x^3 + 30x^2 + 36x$ (i) $7x^2 - 28$

(j) $2x^2 - 10x + 12$ (k) $3x^3 + 21x^2 + 30x$ (l) $6x^3 - 54x$

R2 I can find the roots of a Quadratic Equation by factorising.

1. Solve each of these quadratic equations

(a) $x^2 + 7x + 12 = 0$ **(b)** $x^2 - 4 = 0$ **(c)** $n^2 + 3n + 2 = 0$

(d) $5x^2 + 15x = 0$ **(e)** $p^2 + 11p + 24 = 0$ **(f)** $12a - 3a^2 = 0$

(g) $s^2 + 6s + 8 = 0$ **(h)** $r^2 - 25 = 0$ **(i)** $n^2 + 5n + 6 = 0$

2. Solve each of these quadratic equations

(a) $x^2 - 11x + 24 = 0$ **(b)** $4x^2 - 9 = 0$ **(c)** $n^2 + 3n - 10 = 0$

(d) $5x^2 + 3x = 0$ **(e)** $p^2 - 10p + 24 = 0$ **(f)** $5a^2 - 20 = 0$

(g) $2n^2 + 7n + 3 = 0$ **(h)** $5r^2 + 7r + 2 = 0$ **(i)** $3n^2 - 4n + 1 = 0$

(j) $2r^2 - r - 6 = 0$ **(k)** $6s^2 - 18s - 18 = 6$ **(l)** $7r^2 - 14r = -7$

(m) $n^2 + 8n = -15$ **(n)** $5r^2 - 44r + 120 = -30 + 11r$

R3 I can find the roots of a Quadratic Equation using the Quadratic Formula.

1. Solve these equations giving your answer to 2 significant figures.

(a) $x^2 - 3x - 1 = 0$ **(b)** $2x^2 + 5x + 1 = 0$ **(c)** $5x^2 - 7x - 2 = 0$

2. Solve these equations giving your answer to 3 significant figures.

(a) $3x^2 - 10x = -2$ **(b)** $2x^2 = 6x - 3$ **(c)** $4x^2 + x = 1$

R4 I have revised how to determine where quadratic graphs cut the axes, Turning Points and the Axis of Symmetry.

For each of the quadratic functions given below

(i) Write down the points where the graph of $y = f(x)$ cuts the axes.

(ii) State the equation of the axis of symmetry of $y = f(x)$.

(iii) Write down the coordinates of the turning point of $y = f(x)$.

(iv) Sketch the graph of $y = f(x)$.

1. $f(x) = 5x^2 + 20x$ **2.** $f(x) = x^2 + 6x + 8$

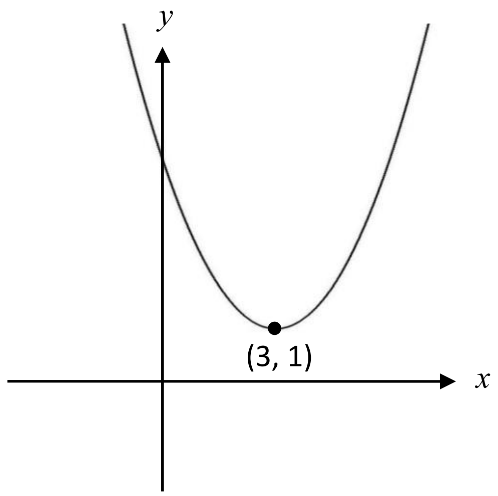
3. $f(x) = 20 - 5x^2$ **4.** $f(x) = 15 - 2x - x^2$

R5	I can determine the equation of a Quadratic from its graph.

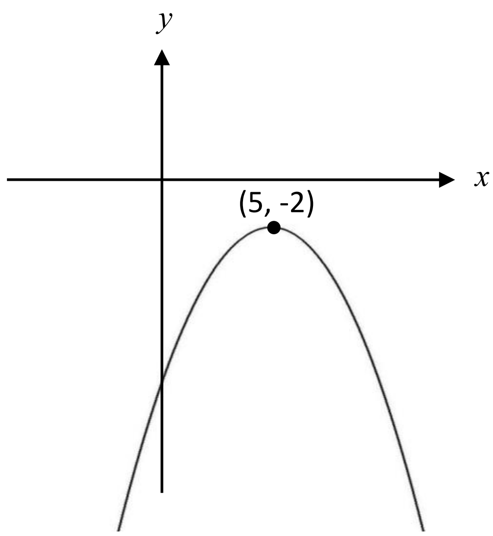
1. The equations of the quadratic functions whose graphs are shown below are of the form $y = (x + a)^2 + b$ or $y = b - (x + a)^2$, where a and b are integers.

Write down the equation of each graph.

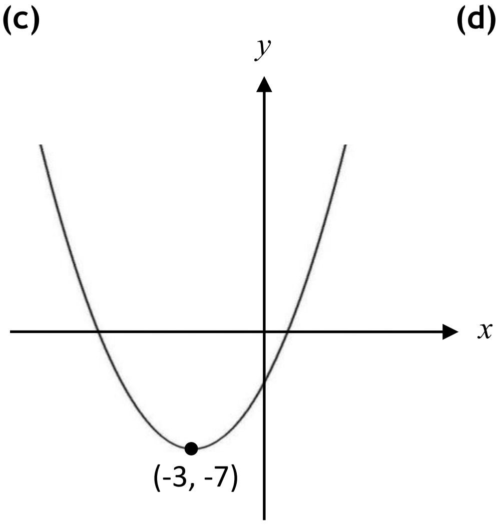
(a)



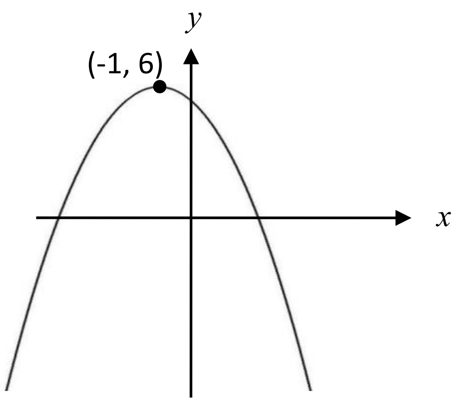
(b)



(c)

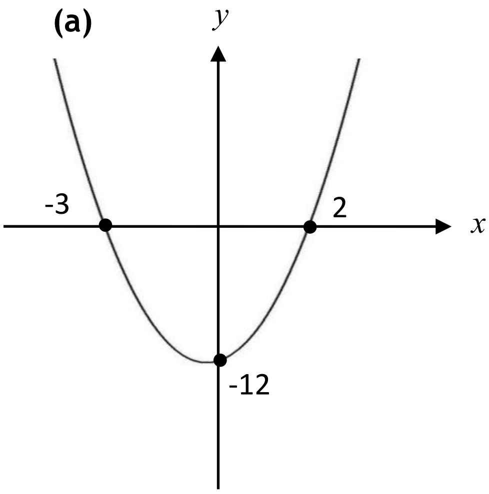


(d)

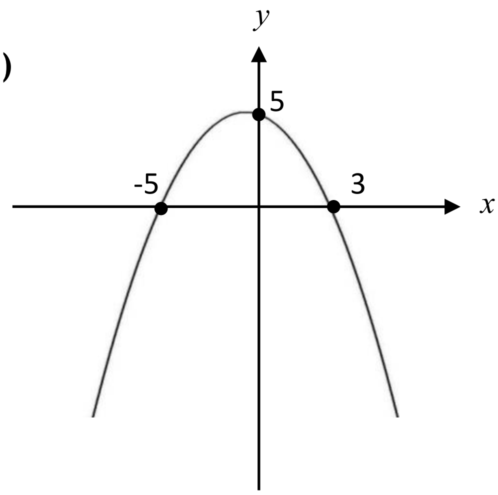


2. Find the equation of each of the parabolas below in the form $y = ax^2 + bx + c$

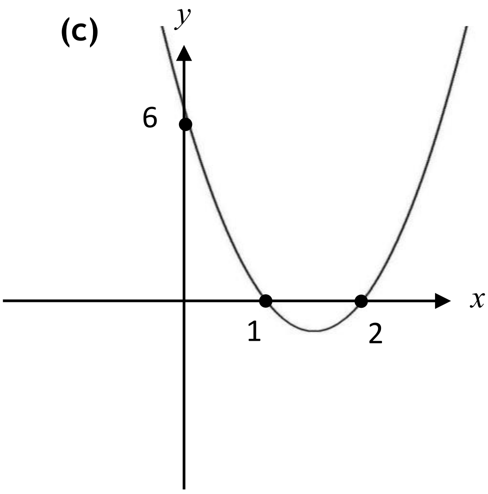
(a)



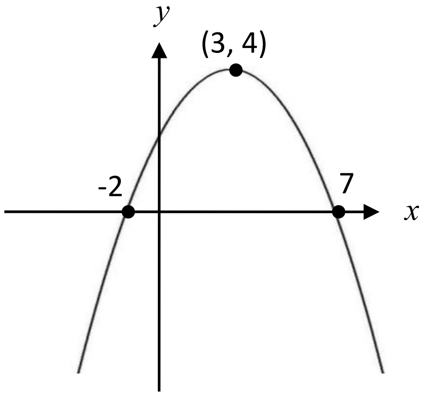
(b)



(c)



(d)



R6 I can solve Quadratic inequalities using a sketch of the graph.

1. Solve the following quadratic inequalities

(a) $(x+2)(x-3) > 0$ **(b)** $x(x-7) < 0$ **(c)** $-x(x-3) \geq 0$

(d) $-(x+4)(x+2) > 0$ **(e)** $(x+5)(x-5) \leq 0$ **(f)** $(x-2)(x-8) > 0$

2. Solve the following quadratic inequalities

(a) $\boldsymbol{x^2 + x - 2} > 0$ **(b)** $2x^2 - 5x - 3 < 0$ **(c)** $3x^2 + 7x + 2 \leq 0$

(d) $\boldsymbol{x^2 - 2} > 0$ **(e)** $2x^2 + 10x \geq 0$ **(f)** $\boldsymbol{x^2 + x} > 0$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can complete the square.

1. **(a)** Show that the function $f(x) = 3x^2 + 30x + 73$ can be written in the form $f(x) = a(x + b)^2 + c$, where a, b and c are constants.

(b) Hence or otherwise find the coordinates of the turning point of function $f(x)$.

(Non-calculator)

2. **(a)** Show the function $f(x) = 9 - 8x - x^2$ can be written in the form $f(x) = p(x + q)^2 + r$ where p, q and r are constants.

(b) Hence or otherwise find the minimum value of $g(x) = \frac{1}{f(x)}$.

(Non-calculator)

3. The cost, c pence of running a car for 20 miles at an average speed of x mph is given by $c = \frac{1}{4}x^2 - 25x + 875$

(a) Express c in the form $p(x - q)^2 + r$

(b) Find the most economical average speed and hence the cost for 20 miles at this speed

4. The height h metres, of a toy rocket is given by $h = 60 + 10t - t^2$ where t seconds is the time of flight

(a) Express h in the form $p(t + q)^2 + r$

(b) Find the maximum height of the rocket and the time taken to reach it

5. **(a)** Show that the function $f(x) = 4x^2 + 16x - 5$ can be written in the form $f(x) = a(x + b)^2 + c$, where a, b and c are constants.

(b) Hence or otherwise, find the coordinates of the turning point of the function f.

(Non-calculator)

6. **(a)** Express $f(x) = 10 - 6x - 3x^2$ in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants.

(b) Find the nature and the coordinates of the turning point of the function.

(Non-calculator)

7. **(a)** Express $f(x) = 2x^2 + 5x - 3$ in the form $f(x) = a(x + p)^2 + q$.

(b) Hence or otherwise sketch the graph of $y = f(x)$.

(Non-calculator)

NR2 I can find the nature of the roots of a Quadratic using the discriminant.

1. **(a)** Determine the nature of the roots of equation $2x^2 + 4x - k = 0$ when $k = 6$.

(b) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.

(Non-calculator)

2. Prove that for all values of k, that the equation $x^2 - 2x + k^2 + 2 = 0$

has no real roots

3. Find the nature of the roots of the equation $(p - 1)^2 + 3p^2 = 6p - 11$.

4. (a) Prove that the roots of the equation,

$(9p^2 - 4qr)x^2 + 2(q + r)x - 1 = 0, where\ p, q, r \in Q$

are real for all values of p, q and r.

(b) Show also that if $q = r$ the roots are rational.

NR3 I can use the discriminant to find an unknown value.

1. Find the value of k for which equation $2x^2 - 3k = 4x^2 + k^2 - 2k$ has equal roots. $k \neq 0$

(Non-calculator)

2. Find the smallest integer value of k for which

$f(x) = (x-2)(x^2 - 2x + k)$ has equal roots.

(Non-calculator)

3. Find the values of k which ensures the following equation has equal roots

$$\frac{(x-3)^2}{x^2+3} = k.$$

4. Find two values of p for which the equation

$$p^2x^2 + 2(p+1)x + 4 = 0$$

has equal roots and solve the equation for x in each case.

5. If the roots of the equation $(x-1)(x+k) = -9$ are equal, find the values of k.

6. Find k, if the roots of the quadratic equation

$$2x^2 + (4k+2)x + 2k^2 = 0$$

are not real.

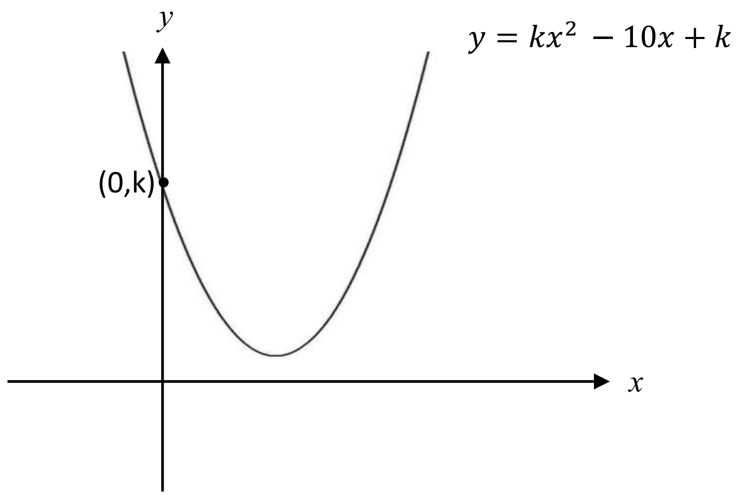
7. Find the range of values of k for which $(k+1)x^2 + 4kx + 9 = 0$ has no real roots.

8. Find the range of values of m for which, $2x^2 + 5mx + m = 0$, has two real and distinct roots.

9. For what range of values of k does the equation $x^2 - 2kx + 2 = k$ have real roots.

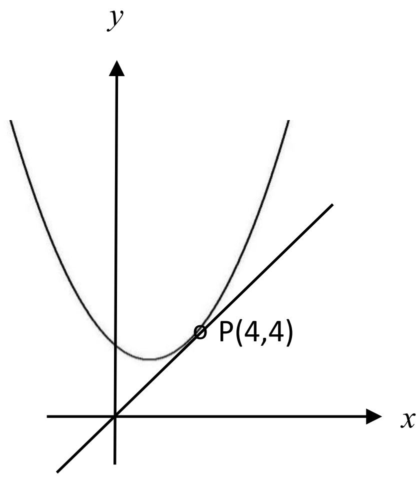
10. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 10x + k$ does not cut the $x-$ axis.

(Non-calculator)



NR4 I can apply the condition for tangency.

1. The point P(4,4) lies on the parabola $y = x^2 + mx + n$



(a) Find a relationship between m and n.

(b) The tangent to the parabola at point P is the line $y = x$.

Find the value of m.

(c) Using your values for m and n, find the value of the discriminant of $x^2 + mx + n = 0$. What feature of the above sketch is confirmed by this value?

2. Show that $y = 17 - 7x$ is a tangent to the parabola $y = -x^2 - x + 8$ and find the point of contact.

(Non-calculator)

3. The line $y = -8x + k$ is a tangent to the parabola $y = 6x - x^2$.

Find the equation of the tangent.

(Non-calculator)

4. **(a)** Show that the line $y = x + 5$ is a tangent to the curve with equation $y = \frac{1}{4}x^2 + 3x + 9$.

(b) Find the point of contact of the tangent to the curve.

NR5 I have experience of cross topic exam standard questions.

1. Figure 1 shows the sketch of the graph $f(x) = (x-3)^2 + 2$.

The graph cuts the y-axis at A and has a minimum turning point at B.

(a) Write down the coordinates of A and B.

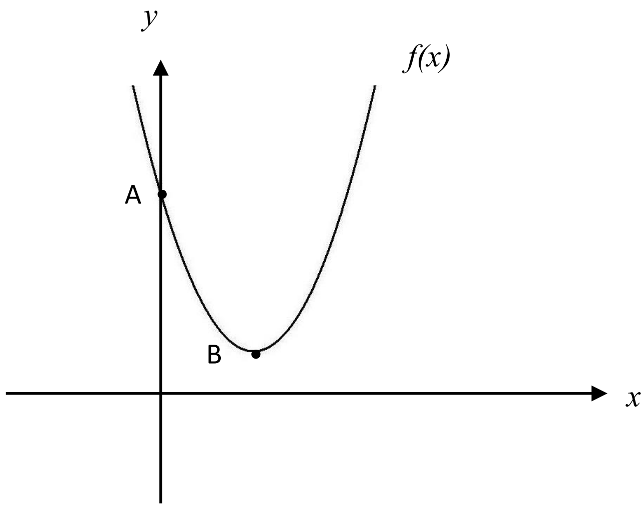


Figure 1

(b) Figure 2 shows the sketch of f(x) and $g(x) = 11 + 6x - x^2$.

Find the area enclosed by the two curves.

(Non-calculator)

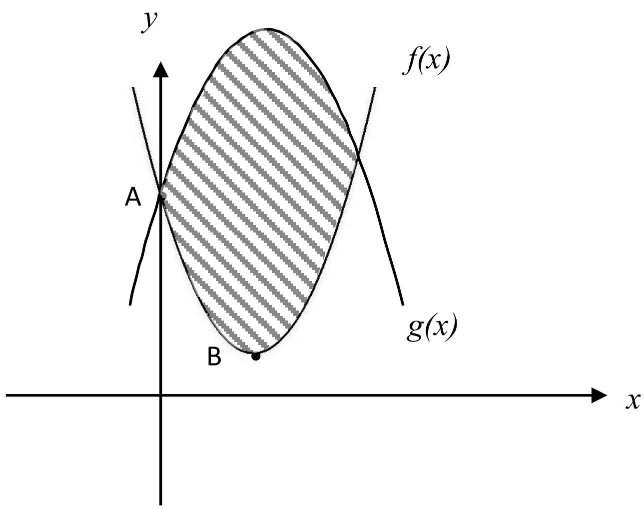
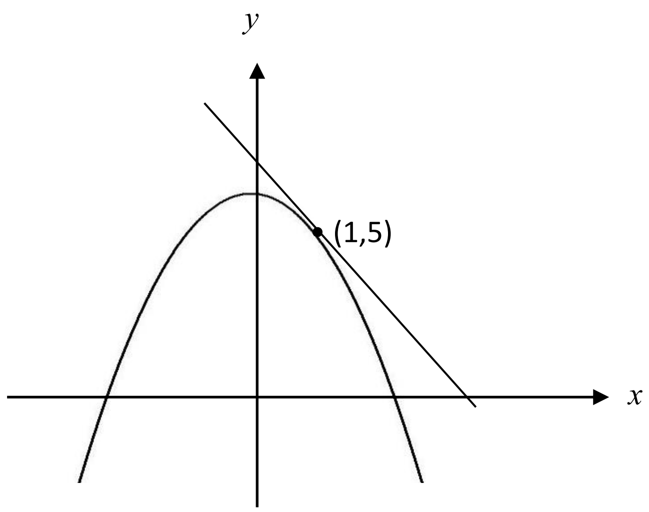


Figure 2

2. **(a)** Find the equation of the tangent to the parabola with equation $y = 7 + x - 2x^2$ at the point (1,5).



(b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 2x - 10y + 26 = 0$.

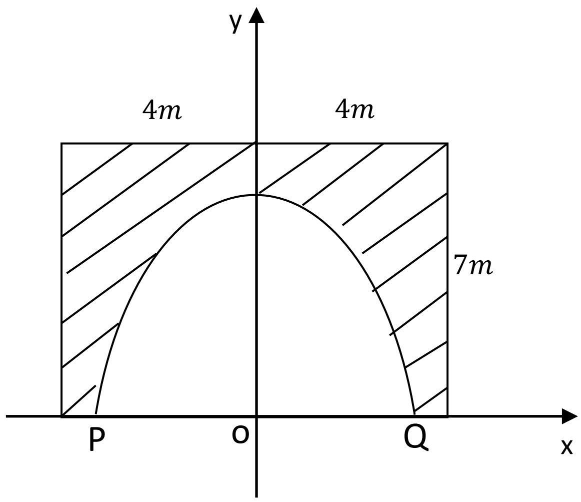
(Non-calculator)

3. **(a)** The points $A(2,6)$, $B(-4,21)$ and $C(7,k)$ are on the same straight line.

Find the value of k.

(b) Find the equation of the tangent to the curve, $y = 2 + 3x - x^2$, at the point where $x = 2$.

4. The concrete on the 7 metre by 8 metre rectangular facing of the entrance to a tunnel is to be re-plastered.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 metre.

The roof is in the form of a parabola with equation $y = 3 - \frac{1}{3}x^2$.

(a) Find the coordinates of the points P and Q.

(b) Calculate the total cost of re-plastering the facing at £8 per square metre.

5. Diagram 1 below shows a rectangular sheet of plastic moulded into a parabolic shape and pegged to the ground to form a cover for a storm shelter. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.

Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres).

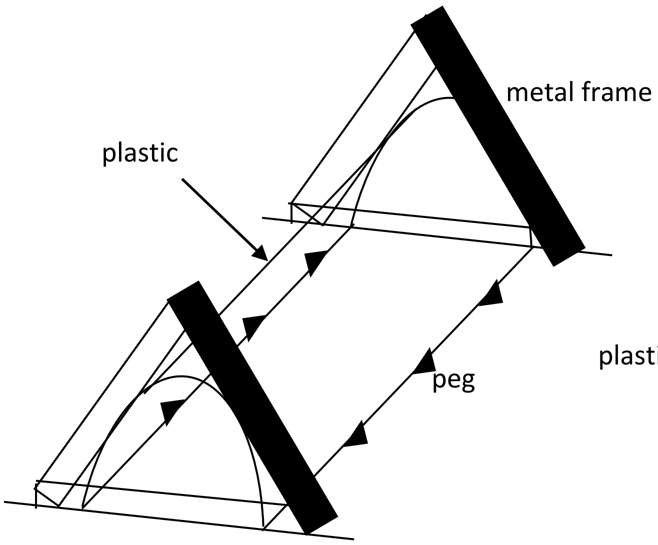


Diagram 1

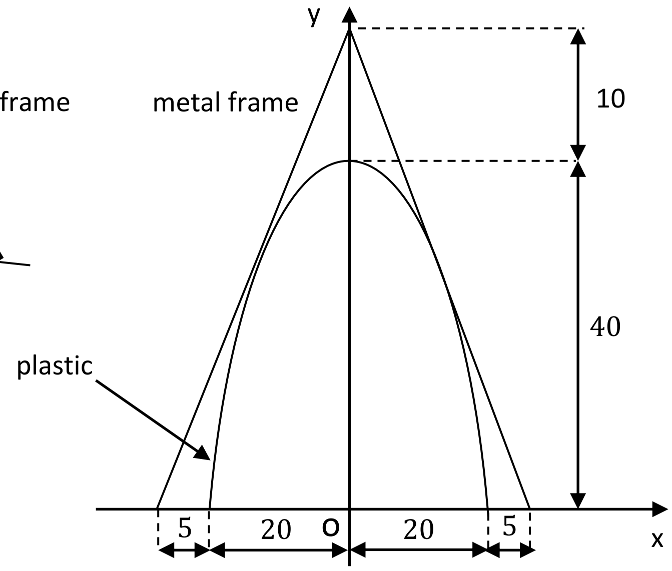


Diagram 2

(a) Show that the equation of the parabolic end is $y = 40 - \frac{x^2}{10}$, $-20 \leq x \leq 20$.

(b) Show that the triangular frame touches the cover without disturbing the parabolic shape.

6. The gradients of two straight lines are the solutions of the quadratic equation, $px^2 - 2x + (2p + 1) = 0$.

Calculate p if,

(i) the lines are perpendicular.

(ii) the lines are parallel.

SOLVING TRIGONOMETRIC EQUATIONS

R7	**I have revised solving basic trigonometric equations in degrees and radians.**

1. Solve the equations:

(a) $5\tan x° - 6 = 2, \quad 0 \leq x \leq 360.$

(b) $7\sin x° + 1 = -5, \quad 0 \leq x \leq 360.$

(c) $4\cos x° + 3 = 0, \quad 0 \leq x \leq 360.$

(d) $3\tan x + 3 = 7, \quad 0 \leq x \leq 2\pi.$

(e) $4\sin x - 2 = -3, \quad 0 \leq x \leq 2\pi.$

(f) $9\cos x - 5 = 0, \quad 0 \leq x \leq 2\pi.$

2. Solve the equations:

(a) $9\tan 2x° - 5 = 3, \quad 0 \leq x \leq 180.$

(b) $4\sin 3x° + 1 = -2, \quad 0 \leq x \leq 360.$

(c) $3\cos 2x° + 2 = 0, \quad 0 \leq x \leq 360.$

3. Solve the equations:

(a) $\tan(x + 30)° = 3, \quad 0 \leq x \leq 360.$

(b) $5\sin(x + 10)° + 3 = -1, \quad 0 \leq x \leq 360.$

(c) $4\cos(x + 26)° + 3 = 0, \quad 0 \leq x \leq 360.$

(d) $\sqrt{3}\tan\left(x + \frac{\pi}{5}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi.$

(e) $6\sin(x + 2) - 2 = 1, \quad 0 \leq x \leq 2\pi.$

(f) $\sqrt{2}\cos\left(x + \frac{\pi}{6}\right) + 1 = 0, \quad 0 \leq x \leq 2\pi.$

NR3	**I can solve trigonometric equations in the context of a problem.**

1. Solve the equation $\sin 2x° - \cos x° = 0$, in the interval $0 \leq x < 180$.

2. Solve the equation $\sin x° - \sin 2x° = 0$, in the interval $0 \leq x < 360$.

3. Solve the equation $3\cos 2x + 10\cos x - 1 = 0$, in the interval $0 \leq x < 2\pi$.

4. Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$, in the interval $0 \leq x < 360$.

5. Solve the equation $2\cos 2x - 5\cos x - 4 = 0$, in the interval $0 \leq x < 2\pi$.

6. Solve the equation $\tan^2 x = 3$, in the interval $0 \leq x < \pi$.

7. Solve the equation $\sin\theta = 4\cos\theta$, in the interval $0 \leq x < 2\pi$.

8. **(a)** Express $3\sin x + 4\cos x$ in the form $k\sin(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

   (b) Hence solve the equation $3\sin x + 4\cos x - 3 = 0$ in the interval $0 \leq x < 2\pi$.

9. **(a)** Express $5\sin x° + 3\cos x°$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

   (b) Hence solve the equation $5\sin x + 3\cos x = 4$ in the interval $0 \leq x < 360$.

10. Two curves have equations $y = 6\cos x°$ and $y = \sin 2x°$.

    Find the coordinates of the points of intersection in the range $0 \leq x < 360$.

11. Two curves have equations $y = -3\cos 2x°$ and $y = \cos x° + 1$.

Find the coordinates of the points of intersection in the range $0 \leq x < 180$.

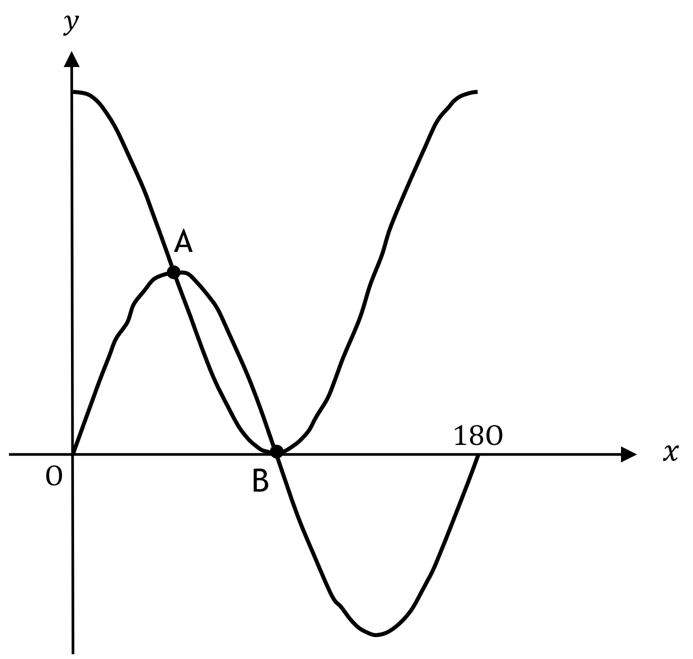
12. A curves has the equation $= \cos 2x° - 3\cos x° + 2$.

Find the coordinates of the points where the curve cuts the x-axis in the range $0 \leq x < 360$.

13. A curves has the equation $= \sin 2x° + \cos x°$.

Find the coordinates of the points where the curve cuts the x-axis in the range $0 \leq x < 360$.

14. The graph shows two curves which have equations $y = 2\cos^2 x°$ and $y = \sin 2x°$ in the range $0 \leq x < 180$.



Find the coordinates of A and B, the points of intersection between the two curves.

<u>DIFFERENTIATION</u>

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1	**I can differentiate algebraic functions which can be simplified to an expression in powers of x, including terms expressed as surds.**

1. Find the derivative of the following

(a) $y = x^3$ **(b)** $y = x^5$ **(c)** $f(x) = x^4$

(d) $y = x^{12}$ **(e)** $f(x) = x^9$ **(f)** $y = x^{14}$

(g) $y = x^8$ **(h)** $y = x^7$ **(i)** $y = x^{24}$

(j) $f(x) = x^{100}$ **(k)** $y = x^{50}$ **(l)** $y = x^{44}$

2. Find the derivative of the following

(a) $y = x^{-2}$ **(b)** $y = x^{-8}$ **(c)** $f(x) = x^{-4}$

(d) $y = x^{-6}$ **(e)** $f(x) = x^{-19}$ **(f)** $y = x^{-5}$

(g) $y = x^{-3}$ **(h)** $y = x^{-10}$ **(i)** $y = x^{-20}$

(j) $f(x) = x^{-100}$ **(k)** $f(x) = x^{-50}$ **(l)** $f(x) = x^{-54}$

3. Find the derivative of the following

(a) $y = 2x^3$ **(b)** $y = 5x^4$ **(c)** $f(x) = 10x^7$

(d) $y = 6x^6$ **(e)** $f(x) = \frac{1}{2}x^6$ **(f)** $y = \frac{2}{3}x^9$

(g) $y = \frac{2}{5}x^{18}$ **(h)** $f(x) = \frac{2}{x^2}$ **(i)** $y = \frac{3}{x^3}$

(j) $y = \frac{25}{x^4}$ **(k)** $y = 7x^3$ **(l)** $y = \frac{3}{x^{-2}}$

(m) $f(x) = \frac{15}{x^{-3}}$ **(n)** $f(x) = 3x^{-5}$ **(o)** $y = \frac{6}{x^{-4}}$

(p) $y = \frac{14}{x^{-4}}$ **(q)** $y = 9x^4$ **(r)** $f(x) = \frac{2}{7x^{-1}}$

(s) $y = \frac{3}{3x^{-5}}$ **(t)** $f(x) = 12x^{-20}$ **(u)** $y = \frac{5}{2x^{-8}}$

4. Find the derivative of the following

(a) $y = x^{\frac{1}{2}}$ **(b)** $f(x) = x^{\frac{2}{3}}$ **(c)** $y = x^{\frac{3}{4}}$

(d) $y = x^{\frac{5}{6}}$ **(e)** $y = x^{\frac{1}{4}}$ **(f)** $f(x) = x^{\frac{2}{5}}$

(g) $f(x) = x^{\frac{6}{7}}$ **(h)** $y = x^{\frac{3}{7}}$ **(i)** $f(x) = x^{\frac{2}{9}}$

(j) $f(x) = x^{\frac{1}{5}}$ **(k)** $f(x) = x^{\frac{3}{11}}$ **(l)** $y = x^{\frac{9}{13}}$

5. Find the derivative of the following

(a) $y = \sqrt{x}$ **(b)** $y = \sqrt{x^3}$ **(c)** $f(x) = \sqrt{x^5}$

(d) $f(x) = \sqrt[3]{x}$ **(e)** $y = \sqrt[3]{x^2}$ **(f)** $y = \sqrt[5]{x^4}$

(g) $f(x) = \frac{1}{\sqrt{x}}$ **(h)** $f(x) = \frac{1}{\sqrt[3]{x^2}}$ **(i)** $y = \frac{1}{\sqrt[3]{x^4}}$

(j) $y = \frac{1}{\sqrt[5]{x^3}}$ **(k)** $y = \frac{2}{\sqrt[3]{x^8}}$ **(l)** $f(x) = \frac{3}{\sqrt[4]{x^3}}$

(m) $y = \frac{1}{2\sqrt[3]{x^2}}$ **(n)** $y = \frac{2}{3\sqrt[4]{x^3}}$ **(o)** $f(x) = \frac{3}{5\sqrt[3]{x^7}}$

6. Find the derivative of the following

(a) $y = x^3 + 3x^2 + 5x$ **(b)** $y = 3x^5 + 2x^4 - x$ **(c)** $y = x^2 + 6x - 1$

(d) $f(x) = x^{\frac{2}{3}} + 4x^2$ **(e)** $f(x) = 3x^{\frac{1}{2}} - 2x^{-5}$ **(f)** $y = 5x^{-2} - 3x^{\frac{1}{2}}$

(g) $f(x) = \frac{1}{2\sqrt[3]{x}} + x^2$ **(h)** $y = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}$ **(i)** $y = \frac{3}{5\sqrt[2]{x^5}} + 5$

(j) $y = \frac{2}{3\sqrt[4]{x^3}} + 2x^2 + x$ **(k)** $y = 5x^2 - \frac{1}{\sqrt[3]{x^2}}$ **(l)** $y = 4x^{-1} - 4x^{\frac{2}{3}}$

(m) $f(x) = 5x^3 - 6x^{-\frac{1}{2}}$ **(n)** $f(x) = 4x^2 + \frac{6}{\sqrt[3]{x}}$ **(o)** $y = x^2 - 5 - \frac{1}{x^2}$

7. Find the derivative of the following

(a) $y = (x+1)(x+2)$ **(b)** $y = (x+2)(x-3)$ **(c)** $y = (x+2)^2$

(d) $y = (x-3)(x+4)$ **(e)** $y = (x-5)(2x-2)$ **(f)** $y = x(x-4)$

(g) $y = (2x-3)(x+4)$ **(h)** $y = x^2(x-2)$ **(i)** $y = \frac{1}{x^2}(x^3+2x)$

(j) $y = \frac{1}{x}(x^2+x)$ **(k)** $y = (\frac{1}{x}+1)^2$ **(l)** $y = \frac{1}{\sqrt{x}}(\frac{1}{\sqrt{x}}-1)$

8. Find the derivative of the following

(a) $y = \frac{x^2+3x+5}{x}$ **(b)** $y = \frac{2x^3+x^2+x}{x}$ **(c)** $y = \frac{x^4-6x+x^3}{x^2}$

(d) $y = \frac{x+5}{x}$ **(e)** $y = \frac{3+x^3}{x^2}$ **(f)** $y = \frac{x+2}{\sqrt{x}}$

(g) $y = \frac{(x+1)(x+2)}{x}$ **(h)** $y = \frac{(x-1)(x+3)}{x^2}$ **(i)** $y = \frac{3x^2+5x+1}{2x^2}$

R2 **I am able to differentiate expressions which contain terms involving $sinx$ and $cosx$, expressed in radians.**

1. Find the derivative of the following

(a) $y = \sin x$ **(b)** $y = \cos x$ **(c)** $f(x) = 2\sin x$

(d) $y = 5\sin x$ **(e)** $y = -2\cos x$ **(f)** $f(x) = -2\sin x$

(g) $y = -6\sin x$ **(h)** $y = \cos x + 2\sin x$ **(i)** $y = 2\sin x - \cos x$

(j) $y = 5\cos \mathrm{x} - 3\sin x$ **(k)** $y = 2\cos x + 4\sin x$ **(l)** $y = 9\sin x + \cos x$

(m) $y = 3\cos \mathrm{x} - 5\sin x$ **(n)** $y = 9\sin x + 7\cos x$ **(o)** $y = \sin x + \cos x$

(p) $y = 3\sin \mathrm{x} - 5\cos x$ **(q)** $y = 9\cos x + 7\sin x$ **(r)** $y = \cos x - \sin x$

R3	**I can differentiate a composite function using the chain rule.**

1. Find the derivative of the following

(a) $y = (x+5)^2$ **(b)** $y = (x-8)^5$ **(c)** $y = (x+2)^3$

(d) $y = (2x-3)^2$ **(e)** $y = (3x-1)^5$ **(f)** $y = (4x+7)^4$

2. Find the derivative of the following

(a) $y = (x+5)^{-2}$ **(b)** $y = (3x-5)^{-3}$ **(c)** $y = (x+2)^{-8}$

(d) $y = \frac{1}{(2x-2)^2}$ **(e)** $y = \frac{1}{(3x+3)^5}$ **(f)** $y = \frac{1}{(6x-1)^4}$

(g) $y = \sqrt{(x+3)}$ **(h)** $y = \sqrt{(x-2)}$ **(i)** $y = \sqrt{(x+2)^3}$

(j) $y = \frac{1}{\sqrt{(x+2)}}$ **(k)** $y = \frac{1}{\sqrt{(x-10)}}$ **(l)** $y = \frac{2}{\sqrt{(x+2)^5}}$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1	**I can determine the equation of a tangent to a curve, at a given point by differentiation.**

1. Find the equation of the tangent to the curve at the given point

(a) $y = x^2$ at (1,4)

(b) $y = x^2 + 2x$ at (0,2)

(c) $y = x^3 + 3$ at (2,8)

(d) $y = x^2 - 6x + 5$ at x = 2

(e) $y = \sqrt{x} + 1$ at x = 1

(f) $y = x^2 + 2x - 3$ at (0,2)

(g) $f(x) = 5x^2 - 6x^{\frac{1}{2}}$ at x = 1

(h) $y = (x + 4)^2$ at (2,-2)

(i) $y = \frac{x^2 + 5x + 10}{x^2}$ at x = 1

(j) $y = \frac{1}{\sqrt{(x+1)}}$ at (3,3)

2. A curve has equation $y = 3x^2 - 4x$. At the point T the tangent to this curve has a gradient of 2, find the coordinates of T and hence the equation of the tangent.

3. A curve has equation $f(x) = 2x^2 + 8x - 3$. A tangent to this curve has a gradient of -4, find the equation of this tangent.

4. A tangent to the equation $y = \frac{2}{\sqrt{x}}$ has a gradient of -1, find the equation of this tangent.

5. A curve has equation $y = x^3 - 6x$. There are two tangents with a gradient 6. Find the equation of both tangents

NR2	I can determine use the stationary points of a curve and state their nature.

1. Find the coordinates of the Stationary points and determine their nature

(a) $y = x^3 - 6x^2 + 9x$

(b) $y = x^3 - 3x^2 - 9x + 12$

(c) $y = x^4 - 4x^3$

(d) $y = x^3 - 3x + 2$

(e) $y = x^3 - 12x + 2$

(f) $y = 2x^3 - 7x^2 + 4x + 1$

(g) $y = 3x^4 + 16x^3$

(h) $y = 8x^3 - x^2 + 11$

(i) $y = \frac{x^3 + 4x + 16}{x}$

(j) $y = (x - 2)(x^2 + 1)$

NR3	I can sketch the graph of an algebraic function by determining stationary points and where it cuts the axes.

1. Sketch the graph of the following functions, stating clearly where it cuts the x & y axis

(a) $y = x^2 - 6x$

(b) $y = x^2 + 5x + 6$

(c) $y = x^2 - 5x$

(d) $y = 2x^3 - 3x^2 - 36x$

(e) $y = (x - 1)^2(x + 2)$

(f) $y = 12 - 4x - x^2$

(g) $y = x^3 - 3x^2$

(h) $y = x(x^2 - 4)$

(i) $y = x^2(2x - 1)$

(j) $y = 3x^4 + 9x^3$

(k) $y = x^3 + 3x^2$

(l) $y = 4x^3 - 2x^4$

(m) $y = 2x^3 - 3x^2$

(n) $y = 3x^2 + 5x - 2$

(o) $y = 2x^2 + 9x + 4$

NR4 I can determine where a function is strictly increasing/decreasing.

1. State whether the function is increasing or decreasing

(a) $y = x^2 - 4x$ at x = 3 **(b)** $y = x^3 - 3x + 2$ at x = -1

(a) $y = x^2 - 10x + 4$ at x = -2 **(b)** $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2$ at x = 4

(a) $y = 4x^2 + 5x + 7$ at x = 1 **(b)** $y = 2x^4 - 4x^2 + 12$ at x = -1

2. For each function state the intervals in which it is increasing AND decreasing

(a) $y = x^2 - 5x + 12$ **(b)** $y = 2x^2 + x + 3$

(a) $y = 8 + 2x - x^2$ **(b)** $y = x^3$

(a) $y = 3x - x^3$ **(b)** $y = x^2(3 - 2x)$

3. Show that the function $y = x^3 - x^2 + x$ is never decreasing

4. Show that the function $y = 2x^5 + 5$ is never decreasing

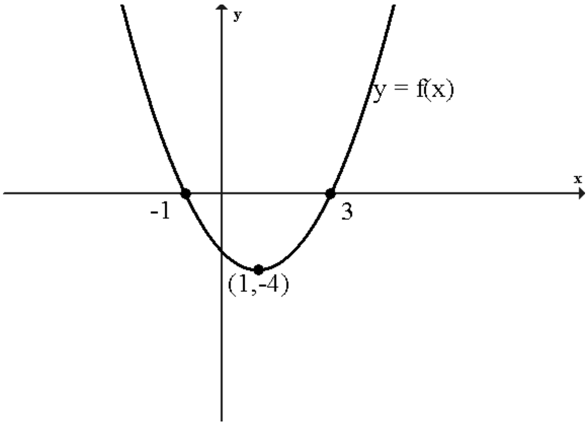
5. Show that the function $y = 12x^2 - 6x - 8x^3$ is never increasing

6. Show that the function $y = -x^3 - 3x^2 - 3x$ is never increasing

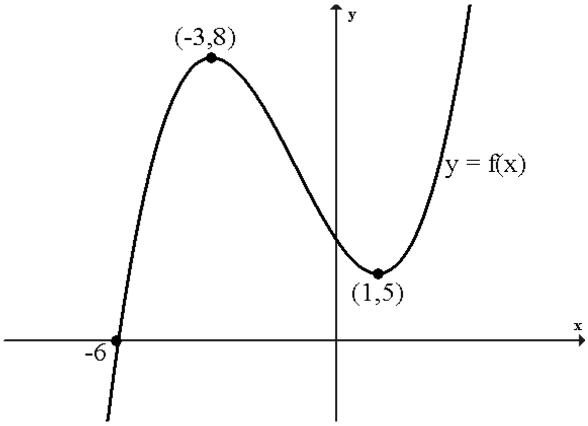
NR5 I can sketch graphs of derivatives.

1. For each graph of f(x) sketch f'(x)

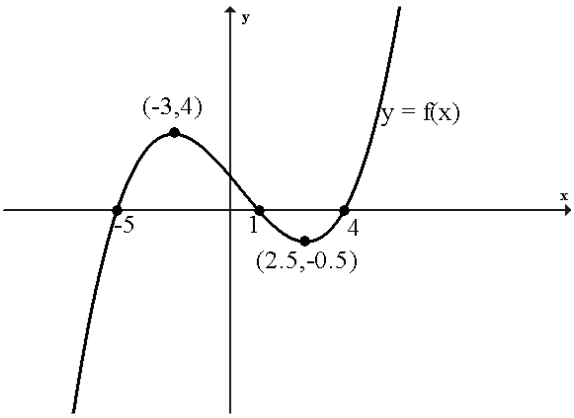
(a)



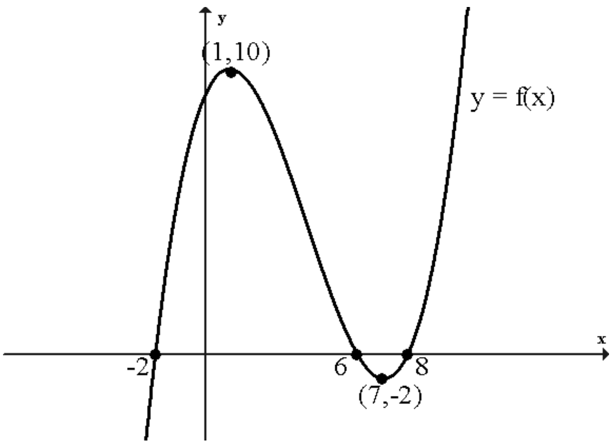
(b)



(c)



(d)



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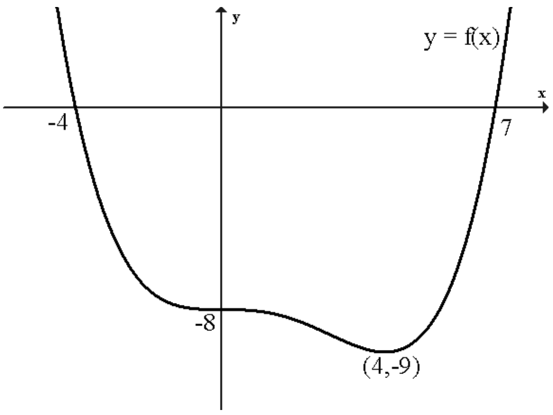

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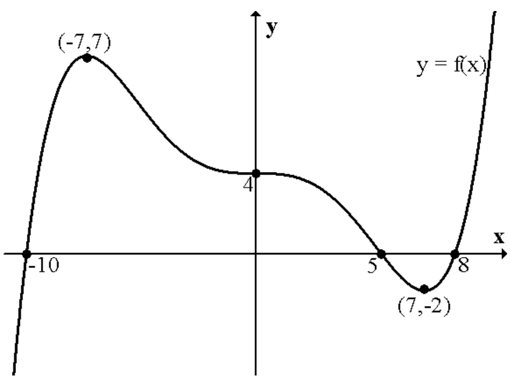

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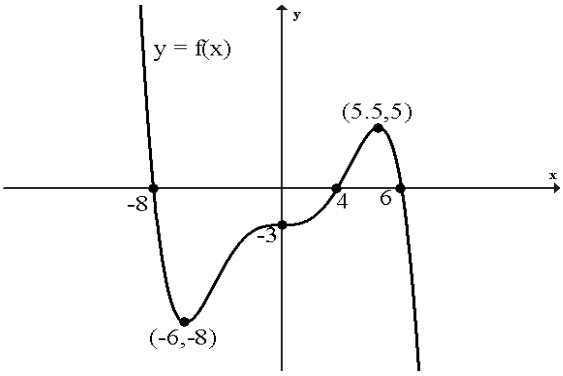

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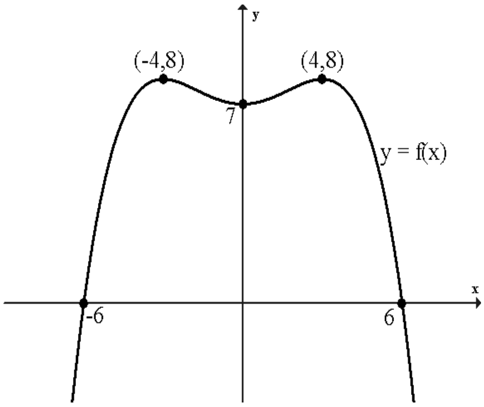


NR8 I can differentiate $sinkx$ and $coskx$ (including applications of the chain rule).

1. Find the derivative of the following

(a) $y = \sin 2x$ **(b)** $y = \cos 2x$ **(c)** $f(x) = 2\sin 3x$

(d) $y = 2\sin 4x$ **(e)** $y = -2\cos 3x$ **(f)** $f(x) = -2\sin 2x$

(g) $y = \sin(2x + 1)$ **(h)** $y = \cos(3x - 2)$ **(i)** $f(x) = 2\sin 2x$

(g) $y = sin^2 x$ **(h)** $y = cos^3 x$ **(i)** $f(x) = \frac{1}{sin^2 x}$

(j) $f(x) = \frac{1}{cos^2 x}$ **(k)** $f(x) = \frac{1}{sin^3 x}$ **(l)** $f(x) = \frac{1}{2sin^3 x}$

(m) $y = \sin x^2$ **(n)** $y = \cos x^2$ **(o)** $y = 2\sin x^3$

NR9 I have experience of cross topic exam standard questions.

1. **(a)** Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$

(b) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}}$

2. **(a)** Given that (x - 1) is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic

(b) Show that the curve with equation $y = x^4 + 4x^3 + 2x^2 - 20x + 3$ has only 1 Stationary point.

Find the x coordinate and determine the nature of this point.

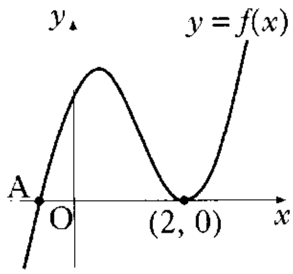
3.

The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.

(*a*) Find the x-coordinate of the maximum turning point.

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$.

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$.



4.

(*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$.

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

5.

A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.

(*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.

6.

Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$.

(*b*) Solve $p'(x) = q'(x)$.

INTEGRATION

Section A

This section is designed to provide examples which develop routine skills necessary for completion of this section.

R1 I can evaluate the definite integral of a polynomial functions with integer limits.

1. Find

(a) $\int_0^1 (x^2 - 3x + 4)\ dx$ **(b)** $\int_0^1 (4x^2 + 3x)\ dx$ **(c)** $\int_0^1 (x^3 + 2x^2 - 1)\ dx$

(d) $\int_0^2 (2x - 1)(x + 2)\, dx$ **(e)** $\int_{-1}^1 2x^2\,(2x + 1)\, dx$ **(f)** $\int_{-2}^1 (2x^3 - x^2 + 3x)\ dx$

2. Find

(a) $\int_{-1}^1 (5x^3 - 2x)\ dx$ **(b)** $\int_{-1}^1 (3x^2 - 4x + 2)\ dx$ **(c)** $\int_{-1}^1 (3x + 2)(x - 2)\, dx$

(d) $\int_0^2 (3x^2 + 8x - 5)\ dx$ **(e)** $\int_{-2}^0 (x - 3)^2\ dx$ **(f)** $\int_{-1}^0 (x^2 - 2x + 7)\ dx$

(g) $\int_0^3 x(x - 2)(x - 3)\, dx$ **(h)** $\int_{-2}^2 (x + 2)(x - 2)\, dx$ **(i)** $\int_1^4 (x - 1)(x - 2)\, dx$

R2 I can evaluate the definite integral of a function with limits in radians, surds or fractions.

1. Evaluate

(a) $\int_0^{\pi} cos2x\ dx$ **(b)** $\int_0^{\pi/2} cos2x\ dx$ **(c)** $\int_0^{\pi} sin2x\ dx$

(d) $\int_0^{\pi/4} sin2x \ dx$ **(e)** $\int_0^{\pi/3} cos3x \ dx$ **(f)** $\int_0^{2\pi} cos\frac{1}{2}x \ dx$

2. Evaluate

(a) $\int_0^{\pi}(\sin t + \cos t) \ dt$ **(b)** $\int_0^{\pi/4} sin4t + cos4t \ dt$

(c) $\int_0^{\pi/4} \cos\left(2t + \frac{\pi}{2}\right) \ dt$ **(d)** $\int_{\pi/6}^{\pi/4} \sin\left(2t - \frac{\pi}{3}\right) \ dt$

3. Evaluate

(a) $\int_0^{1/2}(x^3 + 12x^2 + 7) \ dx$ **(b)** $\int_{-1}^{1/2}(3x^2 - 4x) \ dx$ **(c)** $\int_0^{2/3}(9x^2 + 8) \ dx$

(d) $\int_{-1/2}^{1}(9x^2 + 2x - 1) \ dx$ **(e)** $\int_0^{\sqrt{3}}(2x + 4) \ dx$ **(f)** $\int_1^{\sqrt{3}/2}(10 - 2x) \ dx$

R3 I can apply a standard integral of the form $f(x) = (px + q)^n$ with $n \neq -1$.

1. Find

(a) $\int (x + 2)^8 \ dx$ **(b)** $\int (2x + 4)^3 \ dx$ **(c)** $\int (5x + 7)^4 \ dx$

(d) $\int (2x - 1)^5 \ dx$ **(e)** $\int 6(5 - 4x)^6 \ dx$ **(f)** $\int (10 - x)^{-10} \ dx$

(g) $\int 3(4x + 1)^{-3} \ dx$ **(h)** $\int 2(5x - 9)^{-5} \ dx$ **(i)** $\int (3 - 7x)^{-4} \ dx$

(j) $\int (x - 1)^{\frac{1}{2}} \ dx$ **(k)** $\int (2x - 1)^{\frac{1}{3}} \ dx$ **(l)** $\int (2x - 1)^{\frac{1}{4}} \ dx$

(m) $\int (2x - 2)^{\frac{1}{2}} \ dx$ **(n)** $\int (3x + 4)^{\frac{2}{3}} \ dx$ **(o)** $\int (7 + 3x)^{\frac{3}{4}} \ dx$

2. Find

(a) $\int \frac{1}{(5x+3)^5}\ dx$ **(b)** $\int \frac{dx}{(3x-2)^4}$ **(c)** $\int \frac{3}{(4-2x)^6}\ dx$

(d) $\int \frac{2\ dx}{(x-2)^3}$ **(e)** $\int \frac{3\ dx}{(4x+2)^4}$ **(f)** $\int \frac{1}{(5x-2)^{\frac{1}{2}}}\ dx$

3. Find

(a) $\int \sqrt{4x+2}\ \ dx$ **(b)** $\int 6\sqrt{3x+1}\ \ dx$ **(c)** $\int \sqrt{9-5x}\ \ dx$

(d) $\int \sqrt[3]{2x-3}\ \ dx$ **(e)** $\int \sqrt[3]{6x-2}\ \ dx$ **(f)** $\int \sqrt[4]{2x+4}\ \ dx$

(g) $\int \frac{1}{\sqrt{(3x-4)}}\ dx$ **(h)** $\int \frac{dx}{\sqrt{(x+8)}}$ **(i)** $\int \frac{2dx}{\sqrt{(2x-5)}}$

R4 I can integrate sin^2x and cos^2x by first making a substitution.

1. Find

(a) $\int sin^2x\ \ dx$ **(b)** $\int cos^2x\ \ dx$ **(c)** $\int 2sin^2x\ \ dx$

(d) $\int 2cos^2x\ \ dx$

Section B

This section is designed to provide examples which develop Course Assessment level skills

NR1 I can evaluate one of the limits of a definite integral given the value of the definite integral.

1. Find a, when $a > 0$

(a) $\int_0^a (2x+2)dx = 8$ **(b)** $\int_0^a x^2\ dx = \frac{64}{3}$

2. Given that, $\int_0^a 3x^{1/2}\,dx = 16,$ calculate the value of a.

3. Find a for $0 \le t \le 2\pi$ given:

(a) $\int_0^a \cos t \;\; dt = 1$ **(b)** $\int_0^a \sin t \;\; dt = 2$

4. Given that $\int_0^a 5sin3x \;\; dx = \frac{10}{3},\ 0 \le a \le \pi,$ calculate the value of a.

5. Determine p, given that $\int_1^p x^{1/2}\,dx = 42$

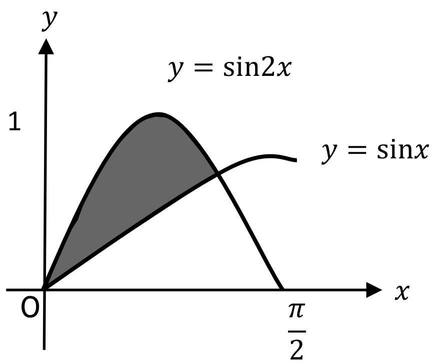
1. Given that $\int_0^k \frac{1}{(4-3x)^2} \;\; dx = \frac{1}{36}$, find k.

NR5 I have experience of cross topic exam standard questions.

Integration and the addition formula

1. **a)** Write down a formula for $\sin 2x$, and use it to solve the equation $\sin 2x = \sin x$ for $0 \le x \le \frac{\pi}{2}$.

   b) Find the shaded area enclosed by the curves $y = \sin 2x$ and $y = \sin x$.



Integration and quadratic graphs

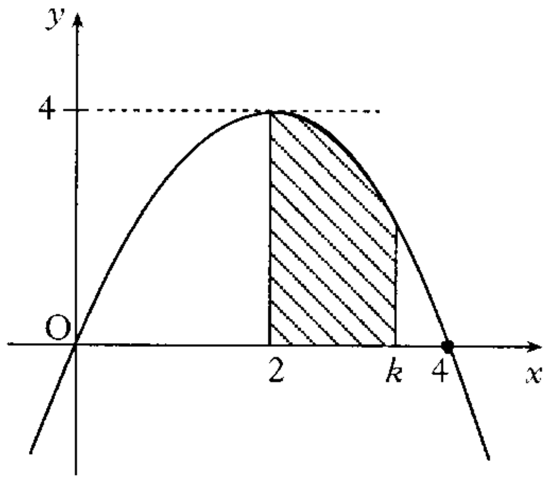
2. The parabola shown crosses the x-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$.

The shaded area is bound by the parabola, the x-axis and the lines $x = 2$ and $x = k$.

(a) Find the equation of the parabola.

(b) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$



Integration and the wave function

3. **(a)** The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \leq a \leq 2\pi$.

Calculate the values of R and a.

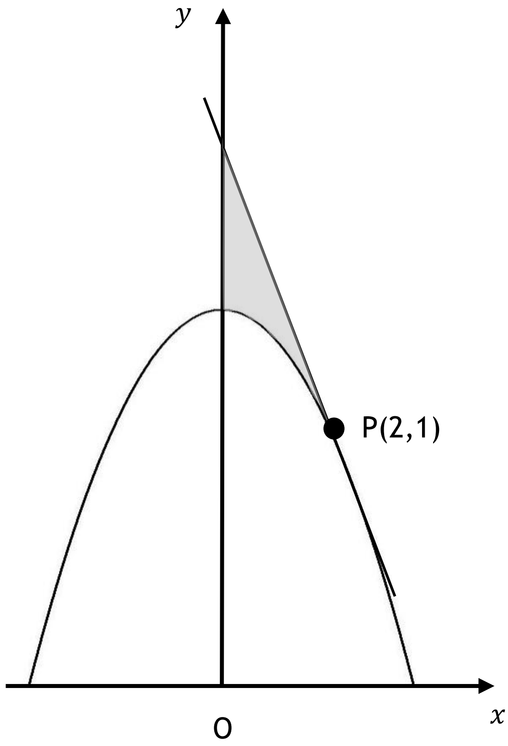
(b) Hence find the value of t, where $0 \leq t \leq 2$, for which

$$\int_0^t (3\sin x - 5\cos x)\, dx = 3$$

Integration and Differentiation

4. **(a)** Find the equation of the tangent to the parabola $y = 5 - x^2$ at P(2,1)

(b) Calculate the area of the shaded region bounded by the tangent, the parabola and the y axis.


y
P(2,1)
O
x

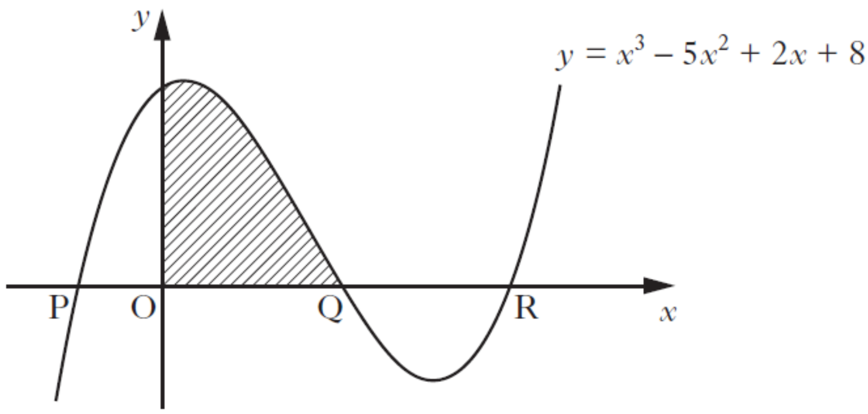
Integration and polynomials

5. **(a)** **(i)** Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

(b) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$



The curve crosses the x-axis at P, Q and R.

Determine the shade area.