The Circle Past Papers Unit 2 outcome 4

Multiple Choice Questions

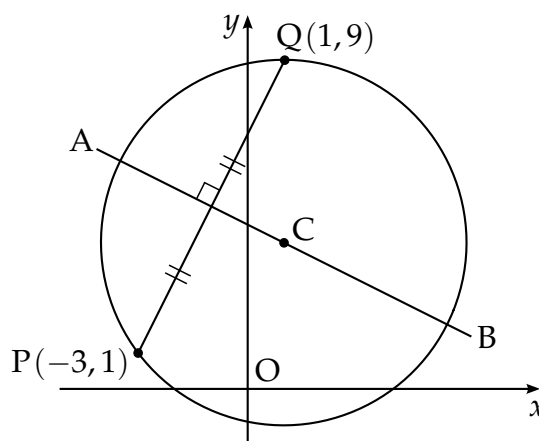*Each correct answer in this section is worth two marks.*

1. A circle has equation
   $(x - 3)^2 + (y + 4)^2 = 20$.

   Find the gradient of the tangent to the circle at the point $(1, 0)$.

   A. -2

   B. $-\frac{1}{2}$

   C. $\frac{1}{2}$

   D. 2

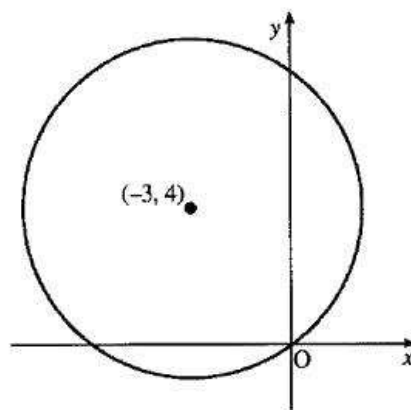*[END OF MULTIPLE CHOICE QUESTIONS]*

Written Questions

[SQA] 2. Find the equation of the tangent at the point $(3, 4)$ on the circle $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

[SQA] 3. Find the equation of the tangent at the point $(3, 1)$ on the circle $x^2 + y^2 - 4x + 6y - 4 = 0$. **5**

[SQA] 4. Find the equation of the circle which has $P(-2, -1)$ and $Q(4, 5)$ as the end points of a diameter. **3**

[SQA] 5. The point $P(2, 3)$ lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

[SQA] 6. The line $y = -1$ is a tangent to a circle which passes through $(0, 0)$ and $(6, 0)$.

Find the equation of this circle. **6**

[SQA] 7. Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$. **5**

[SQA] 8. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

[SQA] 9. Explain why the equation $x^2 + y^2 + 2x + 3y + 5 = 0$ does **not** represent a circle. **2**

[SQA] 10. For what range of values of c does the equation $x^2 + y^2 - 6x + 4y + c = 0$ represent a circle? **3**

[SQA] 11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(a) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(b) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(c) The tangent in (b) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

[SQA] 12. (a) Find the equation of AB, the perpendicular bisector of the line joing the points $P(-3, 1)$ and $Q(1, 9)$. **4**

(b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(c) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**
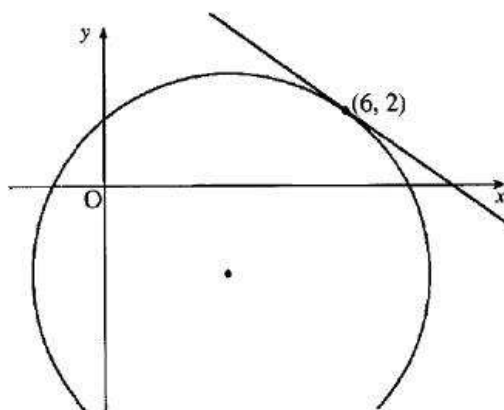
[SQA] 13. Find the equation of the circle with centre (–3, 4) and passing through the origin.

2

[SQA] 14. The circle shown has equation

$$(x-3)^2 + (y+2)^2 = 25.$$

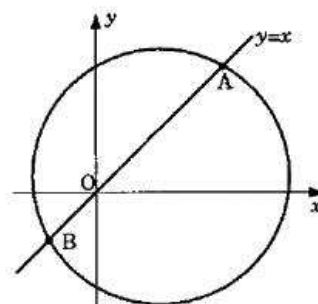Find the equation of the tangent at the point (6, 2).

4

[SQA] 15. The straight line $y = x$ cuts the circle

$x^2 + y^2 - 6x - 2y - 24 = 0$ at A and B.

(a) Find the coordinates of A and B.

3

(b) Find the equation of the circle which has AB as diameter.

3

[SQA]   **16.**   Diagram 1 shows a circle with equation
$x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1,
with equation $y = 2x + 1$.
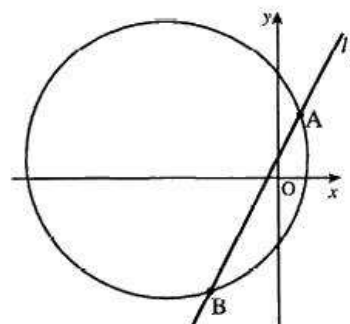The line intersects the circle at A and B.

Diagram 1

(a)   Find the coordinates of the points A and B.   **(5)**

(b)   Diagram 2 shows a second line, l_2, which
passes through the centre of the circle, C,
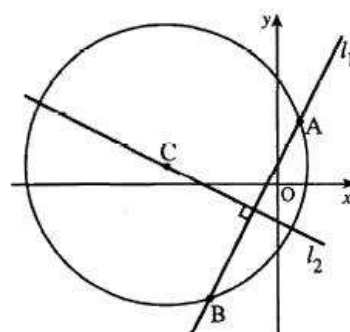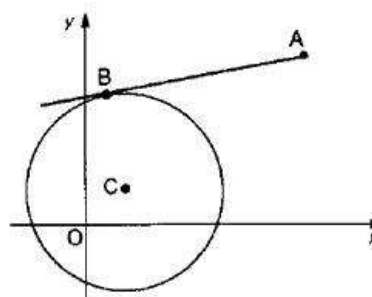and is at right angles to line l_1.

Diagram 2

(i)   Write down the coordinates of C.   **(1)**
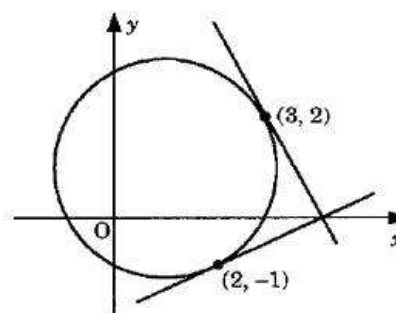(ii)   Find the equation of the line l_2.   **(3)**

[SQA]   **17.**   AB is a tangent at B to the circle with centre C and
equation $(x-2)^2 + (y-2)^2 = 25$.
The point A has co-ordinates (10, 8).
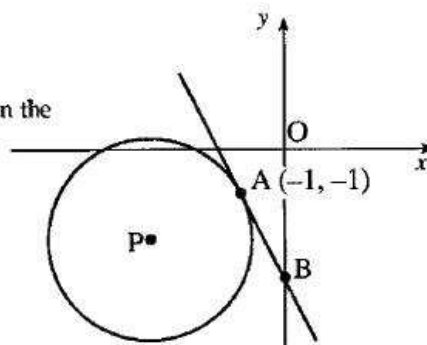Find the area of triangle ABC.

5

[SQA]   **18.**   The circle shown in the diagram has equation
$(x-1)^2 + (y-1)^2 = 5$.
Tangents are drawn at the points (3, 2) and (2, –1).
Write down the coordinates of the centre of the circle and
hence show that the tangents are perpendicular to each other.

4

[SQA] 19. (a) The diagram shows a circle, centre P, with equation

$x^2 + y^2 + 6x + 4y + 8 = 0.$

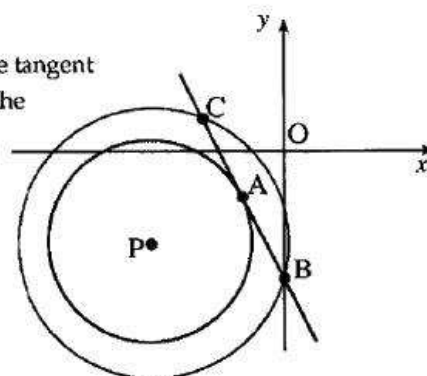Find the equation of the tangent at the point A(–1, –1) on the circle.   4

(b) The tangent crosses the y-axis at B.
Find the coordinates of B.   1

(c) Another circle, centre P, is drawn passing through B. The tangent at A meets the second circle at the point C, as shown in the diagram.
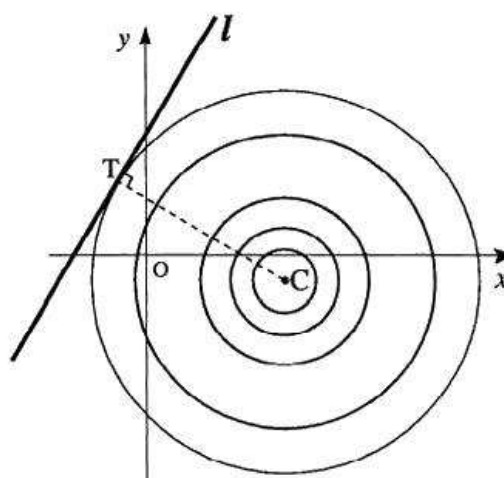Write down the coordinates of C.   1

(d) Find the equation of the circle with BC as diameter.   2

[SQA] 20. In an experiment with a ripple tank, a series of concentric circles with centre C(4,–1) is formed as shown in the diagram.
The line *l* with equation $y = 2x + 6$ represents a barrier placed in the tank.
The largest complete circle touches the barrier at the point T.
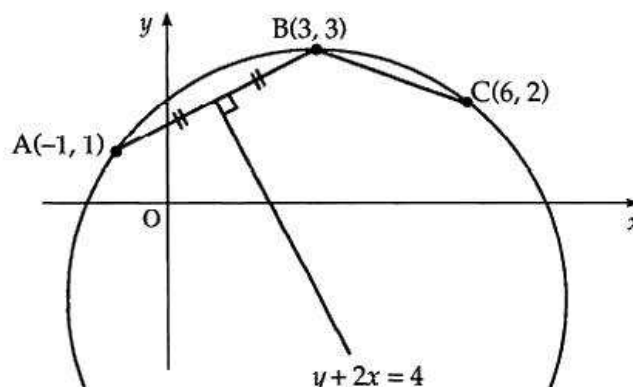
(a) Find the equation of the radius CT.   (3)

(b) Find the equation of the largest complete circle.   (5)

[SQA] **21.** (a) In the diagram, A is the point (−1, 1), B is (3, 3) and C is (6, 2). The perpendicular bisector of AB has equation $y + 2x = 4$. Find the equation of the perpendicular bisector of BC. **(4)**
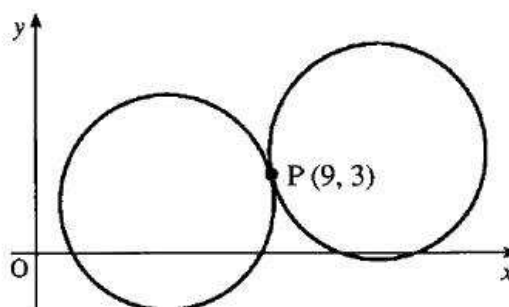

(b) Find the centre and the equation of the circle which passes through A, B and C. **(6)**

[SQA] **22.** Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation $x^2 + y^2 - 10x - 4y + 12 = 0$.
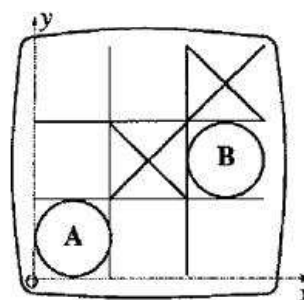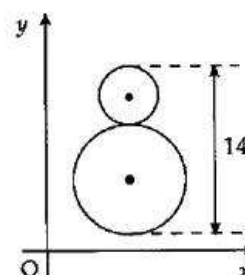
Find the equation of the other circle.

5

[SQA] **23.** This diagram shows a computer-generated display of a game of noughts and crosses.
Relative to the coordinate axes which have been added to the display, the "nought" at A is represented by a circle with equation $(x - 2)^2 + (y - 2)^2 = 4$.

(a) Find the centre of the circle at B.   3
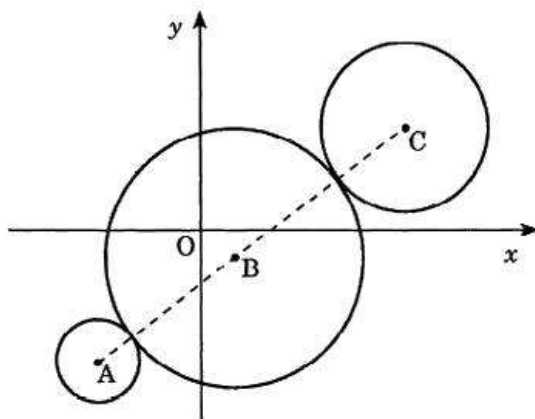(b) Find the equation of the circle at B.   1

[SQA] **24.** A bakery firm makes gingerbread men each 14cm high with a circular "head" and "body".

The equation of the "body" is $x^2 + y^2 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis. Find the equation of the "head".

5

[SQA] 25. When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.

The equations of the circumferences of the outer circles are

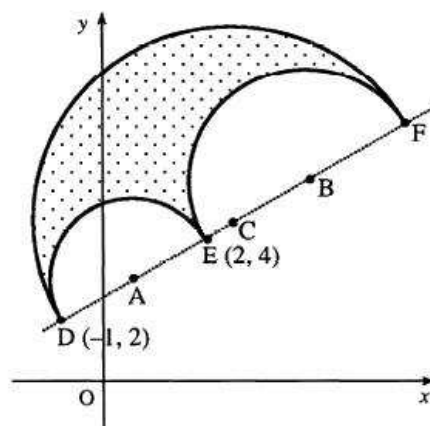$(x+12)^2 + (y+15)^2 = 25$ and $(x-24)^2 + (y-12)^2 = 100$.

Find the equation of the central circle. **(8)**

[SQA] 26. The shape shown in the diagram is composed of 3 semicircles with centres A, B and C which lie on a straight line.

DE is a diameter of one of the semicircles. The coordinates of D and E are (−1, 2) and (2, 4).

(a) Find the equation of the circle with centre A and diameter DE. **(3)**

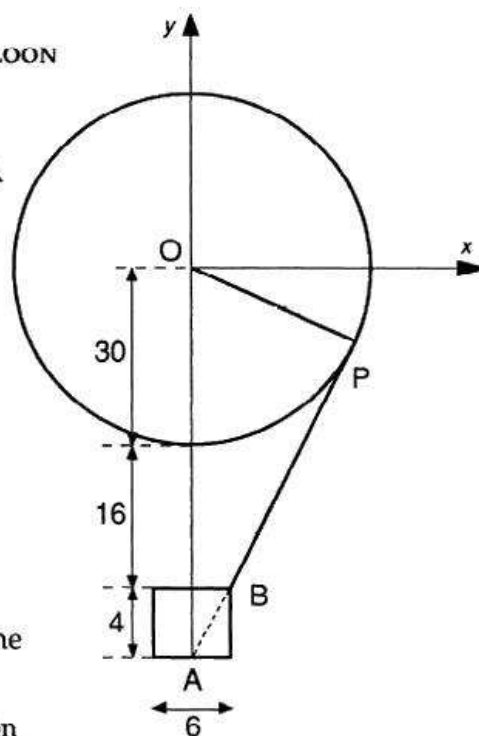The circle with centre B and diameter EF has equation $x^2 + y^2 - 16x - 16y + 76 = 0$.

(b) (i) Write down the coordinates of B.

   (ii) Determine the coordinates of F and C. **(3)**

(c) In the diagram the perimeter of the shape is represented by the thick black line. Show that the perimeter is $5\pi\sqrt{13}$ units. **(3)**

[SQA] 27.

A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.

Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.

(a) Find the equation of the cable PB. **(3)**

(b) State the equation of the circle representing the balloon. **(1)**

(c) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point P. **(5)**

[SQA] 28.

A sports club awards trophies in the form of paperweights bearing the club crest.
Diagram 1 shows the front view of one of these paperweights. Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.
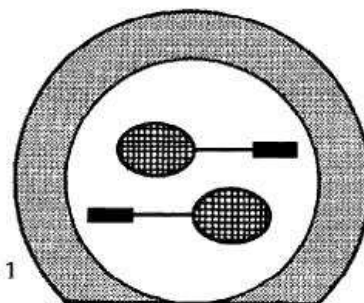
Diagram 1

(a) Relative to x, y coordinate axes, the equation of the outer circle is
$x^2 + y^2 - 8x + 2y - 19 = 0$ and the equation of the base line is $y = -6$.

Show that the equation of the inner circle is $x^2 + y^2 - 8x + 2y - 8 = 0$. **(4)**

(b) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation $3x - 4y + 9 = 0$ as shown in diagram 2.
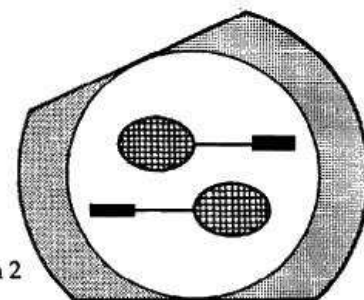Show that this line is a tangent to the inner circle and state the coordinates of the point of contact.
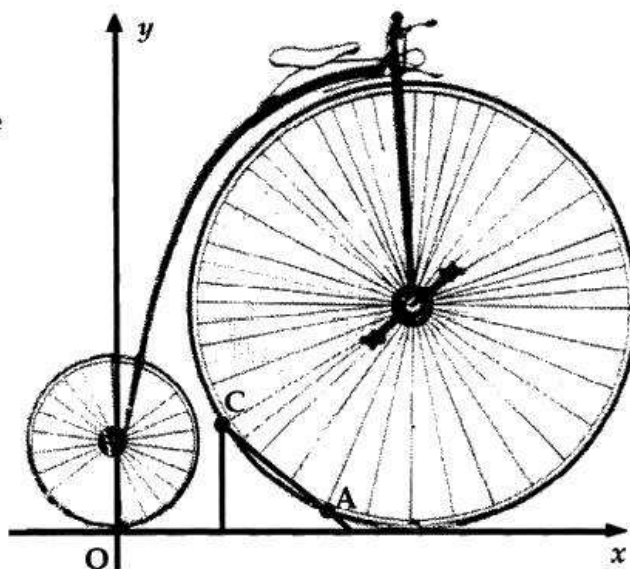
Diagram 2

(7)

[SQA] 29. A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.

With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is

$x^2 + y^2 - 6y = 0$ and

the equation of the front wheel is

$x^2 + y^2 - 28x - 20y + 196 = 0$.

(a) (i) Find the distance between the centres of the two wheels.

    (ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. **(8)**

(b) B(7,3) is half-way between A and C, and P is the centre of the front wheel.

    (i) Find the gradient of PB.

    (ii) Hence find the equation of AC and the coordinates of A and C. **(8)**

[END OF WRITTEN QUESTIONS]