Higher Mathematics

Functions

1. If $f(x) = x^2 + 3x - 4$, find a formula in simplest terms for $f(x - 2)$.

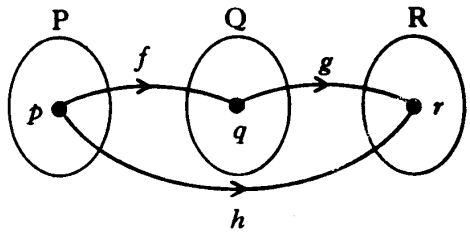
2. Express $H(x) = g(f(x))$ in its simplest form when $f(x) = 4x - 5$ and $g(x) = 2x^2 - x$.

3. The functions f and g are defined by $f : x \to x^2$ and $g : x \to x + 4$.
   Find in its simplest form
   $$f(g(y)) - g(f(y)).$$

4. The diagram shows three functions f , g and h.
   The functions f and g are defined by

   $$f(x) = 3x - 1$$
   and $$g(x) = x^2 + 5$$

   a) If $q = 9$, find the values of p and r.
   b) Find a formula for h(x) in terms of x.



5. If $f(x) = x^2 - 2x + 5$, find formulae , in simplest form , for

   i) $3f(x + 5)$

   ii) $f(y^2) - f(y)$.

6. The function $f(x) = \dfrac{2x + 1}{x + 2}$, where $x \in R$, $x \neq -2$.

   Find a formula for the inverse function of f(x) , stating a suitable domain.

7. Functions f and g are defined by $f(x) = 4x - 3$, $x \in R$ and $g(x) = \dfrac{3x + 4}{2x - 4}$
   $x \in R$, $x \neq 2$.
   a) Find a formula for $H(x) = g(f(x))$.
   b) For which real values of x is the function H undefined ?

8. If $k(x) = \dfrac{1 + 3x}{1 - x}$, $x \in R$, $x \neq 1$, find a formula , in simplest form , for $k(k(x))$.

9. Let $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = 1 - x$ be mappings on the set of real numbers.

   a) Find a formula in its simplest form for the function $K(x) = h(g(f(x)))$.

   b) State , for each of the functions f , g and h , whether there exists an inverse mapping on R , and , where it does , give the formula.

10. The function f is defined by

$$f(x) = 2x + 5 \quad , \quad \text{when } x > 3 \text{ or } x < -1$$
$$f(x) = x^2 - 6 \quad , \quad \text{when } -1 \leq x \leq 3 .$$

   a) Evaluate $f(2) + f(-1)$.
   b) For what values of x is $f(x) = x$?

11. The function $g(x) = \dfrac{1}{x(x+5)}$. For which real values of x is g undefined ?

12. If $f(x) = 2x + 4$ and $g(x) = 5 - 4x$, find a formula for the inverse of $g(f(x))$.

13. The function $H(x) = \dfrac{2x+1}{x-2}$, $x \in R$, $x \neq 2$.

   Show that $H(H(x)) = x$.
   What does this tell you about the function H ?

14. $f : x \to 3x - 2$ and $g : x \to 2x + 6$ are functions from R to R.
   Find in simplest form a formula for $(g(f(x)))^{-1}$.

15. Sketch the graphs of $f(x) = x^3 + 4$, $x \in R$ and $g(x) = x^2 - 1$, $x \in R$.
   Explain why only one of these functions has an inverse.

16. $K(x) = ax^3 + bx$ where a and b are constants.
   If $K(1) = -3$ and $K(3) = 39$ find the constants a and b and hence evaluate $K(5)$.

FUNCTIONS

1) $x^2 - x - 6$

2) $32x^2 - 84x + 55$

3) $8y + 2$

4) $r = 86$, $p = 3\,{}^1/_3$, $9x^2 - 6x + 6$

5) $3x^4 + 24x + 60$, $y^4 - 3y^2 + 2y$

6) $\frac{(2x - 1)}{(2 - x)}$, $x \varepsilon \mathbf{R}$, $x \neq 2$

7) $\frac{12x - 5}{8x - 10}$, $x \varepsilon \mathbf{R}$, $x \neq {}^5/_4$

8) $-\frac{(1 - 2x)}{x}$, $x \varepsilon \mathbf{R}$, $x \neq 0$

ANSWERS

9) Cos^2x, only $h(x)$, $h^{-1}(x) = 1 - x$

10) -7, $x = 3$

11) $x = 0$ or $x = -5$

12) $-\frac{(x + 11)}{8}$

13) Proof, $H(x) = H^{-1}(x)$

14) $\frac{(x - 2)}{6}$

15) Only f(x) has an inverse

16) $a = 2$, $b = -5$, 225