

A mixture of examples (functions).

1. If $f(x) = x^2 - 1$ and $g(x) = x + 1$ find formulas for $f(g(x))$ and $g(f(x))$ and hence solve the equation $f(g(x)) - g(f(x)) = 7$

2. If $f(x) = x + 3$ and $g(x) = 2x^2 - x + 1$ find $f(g(x))$ and $g(f(x))$. Find the values of x for which $f(g(x)) = g(f(x))$

3. If $f(x) = \frac{2x+5}{7x-3}$, find $f^{-1}(x)$.

4. If $f(x) = 2x^2 + 1$, $x \geq 0$, $x \in \mathbb{R}$ find $f^{-1}(x)$

5. $f(x) = x^2 + x + 3$ and $g(x) = x + 1$

Find i) $f(g(x))$ ii) $g(f(x))$ in simplest form

Hence solve the equation $2f(g(x)) = g(f(x))$

6. If $f(x) = 1.5x + 5$, find $f^{-1}(x)$ and hence find i) $f^{-1}(-2)$ ii) $f^{-1}(0)$

7. If $f(x) = \frac{1-2x}{3x}$ state a suitable domain for f .

Find the inverse function $f^{-1}(x)$. For what values of x is $f^{-1}(x)$ undefined?

8. If $f(x) = \frac{2}{\sqrt{3x-2}}$, $x > 2/3$, find a formula for $f^{-1}(x)$ and state a suitable domain for $f^{-1}(x)$.

9. Two functions f and g are given by $f(x) = 4x - 3$ and $g(x) = x^2 - 5x + 14$.

i) Find a formula for $g(f(x))$ giving your answer in the form $ax^2 + bx + c$.

ii) If $g(f(x)) = f(g(x))$, find x .

10. Given that $f(x) = 3x - 2$ and $g(x) = x + 1$, obtain a formula for $h(x) = f(g(x))$.

Hence find $h^{-1}(x)$ and then solve $[h^{-1}(x)]^2 - 1 = 0$

Solutions.

1. $f(g(x)) = x^2 + 2x$ $g(f(x)) = x^2$ $x = 3.5$

2. $f(g(x)) = 2x^2 - x + 4$ $g(f(x)) = 2x^2 + 11x + 16$ $x = -1$

3. $f^{-1}(x) = \frac{3x+5}{7x-2}$ $D = \{x : x \neq \frac{3}{7}, x \in \mathbb{R}\}$ $R = \{x : x \neq \frac{3}{7}, x \in \mathbb{R}\}$

4. $f^{-1}(x) = \sqrt{\frac{x+1}{2}}$

5. $f(g(x)) = x^2 + 3x + 5$ $g(f(x)) = x^2 + x + 4$ $x = -4, -1$

6. $f^{-1}(x) = \frac{2(x-5)}{3}$ $f^{-1}(-2) = -4\frac{2}{3}$ $f^{-1}(0) = -3\frac{1}{3}$

7. $f^{-1}(x) = \frac{1}{3x+2}$ $f^{-1}(x)$ is undefined at $x = -2/3$

8. $f^{-1}(x) = \frac{2(x^2+2)}{3x^2}$ $D = \{x : x > 0, x \in \mathbb{R}\}$

9. i) $g(f(x)) = 16x^2 - 44x + 38$ $f(g(x)) = 4x^2 - 22x + 53$

ii) $x = \frac{5}{2}, -\frac{1}{2}$

10. $h(x) = 3x + 1$; $h^{-1}(x) = \frac{x-1}{3}$ $x = 4, -2$