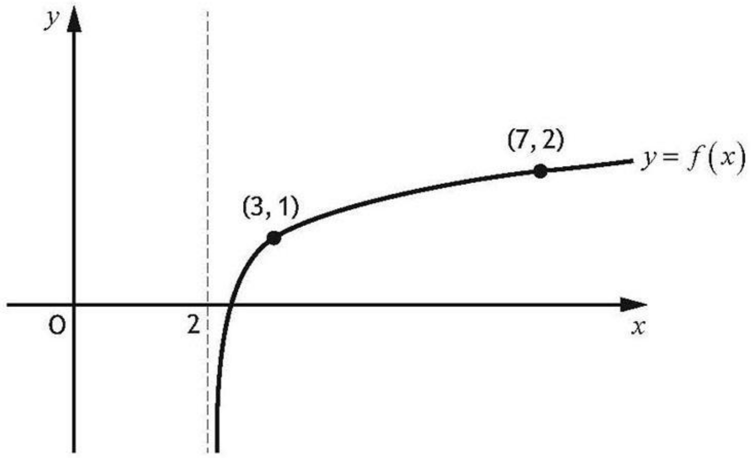
INVERSE FUNCTIONS

QB	$f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = \frac{1}{4}(5 - x)$. (*a*) Find a formula for $k(x)$ where $k(x) = f(g(x))$. (*b*) Find a formula for $h(k(x))$. (*c*) What is the connection between the functions h and k?	 2 2 1
2015 P1 Q5	A function g is defined on $\mathbb{R}$, the set of real numbers, by $g(x) = 6 - 2x$. (a) Determine an expression for $g^{-1}(x)$. (b) Write down an expression for $g(g^{-1}(x))$.	 2 1
2016 P1 Q6	Functions f and g are defined on $\mathbb{R}$, the set of real numbers. The inverse functions f^{-1} and g^{-1} both exist. (a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$. (b) If $g(2) = 7$, write down the value of $g^{-1}(7)$.	 3 1
2017 P1 Q6	A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$. Determine an expression for $h^{-1}(x)$.	 3
2018 P1 Q2	A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by $$g(x) = \frac{1}{5}x - 4.$$ Find the inverse function, $g^{-1}(x)$.	 3
SP	Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$. (a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. (b) If $h(k(x)) = x$, find an expression for $h(x)$.	 2 3
2019 P2 Q8	A function, f, is given by $f(x) = \sqrt[3]{x} + 8$. The domain of f is $1 \le x \le 1000$, $x \in \mathbb{R}$. The inverse function, f^{-1}, exists. (a) Find $f^{-1}(x)$. (b) State the domain of f^{-1}.	 3 1

2021 P1 Q17	A logarithmic function, f, is defined for $x > 2$. The diagram shows the graph of $y = f(x)$.  The inverse function, $f^{-1}(x)$, exists. (a) On the diagram in your answer booklet, sketch the graph of the inverse function. **2** (b) Given that $f(x) = \log_5(x-2)+1$, find the coordinates of the point where the graph of $f^{-1}(x)$ crosses the y-axis. **2**
2021 P1 Q3	A function $f(x)$ is defined on $\mathbb{R}$, by $$f(x) = \frac{x+3}{2}.$$ Find the inverse function, $f^{-1}(x)$. **3**
2022 P1 Q3	A function, h, is defined by $h(x) = 4 + \frac{1}{3}x$, where $x \in \mathbb{R}$. Find the inverse function, $h^{-1}(x)$. **3**

INVERSE FUNCTIONS

<table>
<tr><td>QB</td><td>
<table>
<tr><td>•1</td><td>$f(3-2x)$</td></tr>
<tr><td>•2</td><td>$5-4x$</td></tr>
<tr><td>•3</td><td>$h(5-4x)$</td></tr>
<tr><td>•4</td><td>x</td></tr>
<tr><td>•5</td><td>inverse of each other</td></tr>
</table>
</td></tr>
<tr><td>2015
P1
Q5</td><td>
<table>
<tr><td>•1 let $y=6-2x$ and rearrange.</td><td>•1 $x=\frac{6-y}{2}$ or $y=\frac{6-x}{2}$</td></tr>
<tr><td>•2 state expression.</td><td>•2 $g^{-1}(x)=\frac{6-x}{2}$ or $3-\frac{x}{2}$ or $\frac{x-6}{-2}$</td></tr>
<tr><td>Method 2</td><td>Method 2</td></tr>
<tr><td>•3 equates composite function to x</td><td>$g(g^{-1}(x))=x$ this gains •3</td></tr>
<tr><td>•1 start to rearrange.</td><td>$6-2g^{-1}(x)=x$</td></tr>
<tr><td>•2 state expression.</td><td>$g^{-1}(x)=\frac{6-x}{2}$ or $3-\frac{x}{2}$ or $\frac{x-6}{-2}$</td></tr>
</table>
</td></tr>
<tr><td>2016
P1
Q6</td><td>
<table>
<tr><td></td><td>Method 1:</td></tr>
<tr><td>•1 equate composite function to x</td><td>•1 $f\left(f^{-1}(x)\right)=x$</td></tr>
<tr><td>•2 write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$</td><td>•2 $3f^{-1}(x)+5=x$</td></tr>
<tr><td>•3 state inverse function</td><td>•3 $f^{-1}(x)=\frac{x-5}{3}$</td></tr>
<tr><td></td><td>Method 2:</td></tr>
<tr><td>•1 write as $y=3x+5$ and start to rearrange</td><td>•1 $y-5=3x$</td></tr>
<tr><td>•2 complete rearrangement</td><td>•2 $x=\frac{y-5}{3}$</td></tr>
<tr><td>•3 state inverse function</td><td>•3 $f^{-1}(x)=\frac{x-5}{3}$</td></tr>
<tr><td></td><td>Method 3</td></tr>
<tr><td>•1 interchange variables</td><td>•1 $x=3y+5$</td></tr>
<tr><td>•2 complete rearrangement</td><td>•2 $\frac{x-5}{3}=y$</td></tr>
<tr><td>•3 state inverse function</td><td>•3 $f^{-1}(x)=\frac{x-5}{3}$</td></tr>
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2017 P1 Q6	

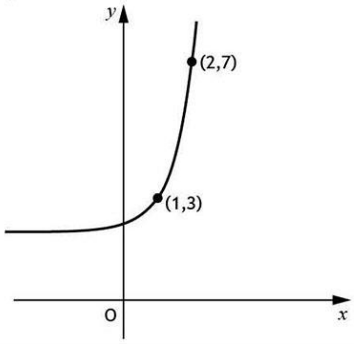
Method 1	Method 1
•¹ equate composite function to x	•¹ $h\left(h^{-1}(x)\right)=x$
•² write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•² $\left(h^{-1}(x)\right)^3+7=x$
•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$
Method 2	**Method 2**
•¹ write as $y=x^3+7$ and start to rearrange	•¹ $y-7=x^3$
•² complete rearrangement	•² $x=\sqrt[3]{y-7}$
•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$
Method 3	**Method 3**
•¹ interchange variables	•¹ $x=y^3+7$
•² complete rearrangement	•² $y=\sqrt[3]{x-7}$
•³ state inverse function	•³ $h^{-1}(x)=\sqrt[3]{x-7}$ or $h^{-1}(x)=(x-7)^{\frac{1}{3}}$

2018 P1 Q2	

15. (a)	•¹ interpret notation •² complete process	•¹ $g\left(x^3-1\right)$ •² $3x^3-2$	**2**
15. (b)	•³ start to rearrange for x •⁴ rearrange •⁵ state expression for $h(x)$	•³ $3x^3=y+2$ •⁴ $x=\sqrt[3]{\frac{y+2}{3}}$ •⁵ $h(x)=\sqrt[3]{\frac{x+2}{3}}$	**3**

SP	**Method 1** $\bullet^1$ equate composite function to x — $\bullet^1$ $g\left(g^{-1}(x)\right)=x$ $\bullet^2$ write $g(g^{-1}(x))$ in terms of $g^{-1}(x)$ — $\bullet^2$ $\frac{1}{5}g^{-1}(x)-4=x$ $\bullet^3$ state inverse function — $\bullet^3$ $g^{-1}(x)=5(x+4)$ **Method 2** $\bullet^1$ write as $y=\frac{1}{5}x-4$ and start to rearrange — $\bullet^1$ $y+4=\frac{1}{5}x$ $\bullet^2$ express x in terms of y — $\bullet^2$ eg $x=5(y+4)$ or $x=\frac{(y+4)}{\frac{1}{5}}$ $\bullet^3$ state inverse function — $\bullet^3$ $g^{-1}(x)=5(x+4)$ **Method 3** $\bullet^1$ interchange variables — $\bullet^1$ $x=\frac{1}{5}y-4$ $\bullet^2$ express y in terms of x — $\bullet^2$ eg $y=5(x+4)$ or $y=\frac{(x+4)}{\frac{1}{5}}$ $\bullet^3$ state inverse function — $\bullet^3$ $g^{-1}(x)=5(x+4)$
2019 P2 Q8	**Method 1** $\bullet^1$ equate composite function to x — $\bullet^1$ $f\left(f^{-1}(x)\right)=x$ $\bullet^2$ write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$ — $\bullet^2$ $\sqrt[3]{f^{-1}(x)}+8=x$ $\bullet^3$ state inverse function — $\bullet^3$ $f^{-1}(x)=(x-8)^3$ **Method 2** $\bullet^1$ write as $y=f(x)$ and start to rearrange — $\bullet^1$ $y=f(x)\Rightarrow x=f^{-1}(y)$ $y-8=\sqrt[3]{x}$ $\bullet^2$ express x in terms of y — $\bullet^2$ $x=(y-8)^3$ $\bullet^3$ state inverse function — $\bullet^3$ $f^{-1}(y)=(y-8)^3$ $\Rightarrow f^{-1}(x)=(x-8)^3$

2021 P1 Q17	(see table below)

17.	(a)		•¹ in first quadrant, graph reflected in $y=x$ passing through $(1,3)$ and $(2,7)$ •² in second quadrant, graph approaching $y=2$ from above	•¹, •² 	**2**
	(b)		**Method 1** •³ interpret information and start to solve •⁴ solve and state coordinates **Method 2** •³ determine $f^{-1}(x)$ •⁴ evaluate $f^{-1}(0)$ and state coordinates	**Method 1** •³ $\log_5(x-2)+1=0$ leading to $\log_5(x-2)=-1$ •⁴ $\left(0,\frac{11}{5}\right)$ **Method 2** •³ $5^{(x-1)}+2$ •⁴ $\left(0,\frac{11}{5}\right)$	**2**

2021 P1 Q3

3.			**Method 1** •¹ equate composite function to x •² write $f\left(f^{-1}(x)\right)$ in terms of $f^{-1}(x)$ •³ state inverse function	**Method 1** •¹ $f\left(f^{-1}(x)\right)=x$ •² $\frac{f^{-1}(x)+3}{2}=x$ •³ $f^{-1}(x)=2x-3$	**3**
			Method 2 •¹ write as $y=f(x)$ and start to rearrange •² express x in terms of y •³ state inverse function	**Method 2** •¹ $y=f(x)\Rightarrow x=f^{-1}(y)$ $2y=x+3$ •² $x=2y-3$ •³ $f^{-1}(y)=2y-3$ $\Rightarrow f^{-1}(x)=2x-3$	

2022 P1 Q3	

Method 1	Method 1
•¹ equate composite function to x	•¹ $h\left(h^{-1}(x)\right)=x$
•² write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•² $4+\frac{1}{3}h^{-1}(x)=x$
•³ state inverse function	•³ $h^{-1}(x)=3(x-4)$
Method 2	**Method 2**
•¹ write as $y=h(x)$ and start to rearrange	•¹ $y=h(x)\Rightarrow x=h^{-1}(y)$ $y-4=\frac{1}{3}x$ or $3y=12+x$
•² express x in terms of y	•² $x=3(y-4)$
•³ state inverse function	•³ $h^{-1}(y)=3(y-4)$ $\Rightarrow h^{-1}(x)=3(x-4)$